

MATH M118: Finite Mathematics
Practice Department Final Examination

1. Write the following argument in symbolic notation. Then construct a truth table and determine whether the argument is valid or not.

If it rains, then the crops will grow. It did not rain. Therefore, the crops did not grow.

Use p : *It rains.* q : *The crops will grow.*

2. Complete the following truth table.

p	q	$p \vee q$	$\neg(p \vee q)$	$p \rightarrow \neg(p \vee q)$
T	T			
T	F			
F	T			
F	F			

3. Complete the following truth table.

p	q	$\neg p$	$\neg p \wedge q$	$q \leftrightarrow (\neg p \wedge q)$
T	T			
T	F			
F	T			
F	F			

4. Let p denote the statement “Jack gets a job,” and q denote the statement “Jack buys a new car.” Write each of the following compound statements using symbolic notation.

- a. Jack did not get a job but he bought a new car.
- b. If Jack gets a job, then he will buy a new car.
- c. Jack did not get a job or buy a new car.
- d. The contrapositive of the statement in 4b.

5. Let $U = \{1, 2, 3, \dots, 10, 11, 12\}$. Let X , Y and Z be subsets of U such that $X = \{2, 4, 6, 8, 10\}$ $Y = \{4, 5, 6, 7, 8\}$ $Z = \{5, 6, 9, 10\}$

a. Find $(X - Y) \cup Z$

b. Find $X' \cap Z$

6. Determine if each of the following statements is true (always true) or false (not always true):

a. $B \cup B' = U$

b. $U - A = A'$

c. $(G \cup H)' = G' \cup H'$

d. $E - F = E \cap F'$

7. Of the 200 students who participated in a survey, 105 like pizza, 75 like tacos and 42 like both. How many like neither? Draw a Venn diagram for this problem.

8. A group consists of 10 men and 11 women. In how many ways can a team of exactly 4 men and 6 women be selected?

9. A car show has 22 cars competing for "Best in Show." Prizes are awarded for first, second and third place; any one car may only win one prize. In how many ways can the prizes be awarded?

10. How many different committees of 3 can be formed from 15 Republicans and 12 Democrats if at least one Republican and at least one Democrat must be on the committee? Circle the correct set up.

a. $P(15,3) P(12,3)$

b. $C(15,3) C(12,3)$

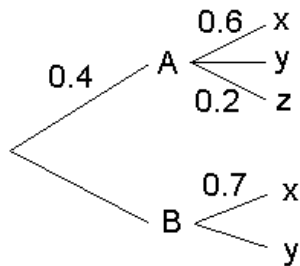
c. $P(15,1)P(12,2) + P(15,2) P(12,1)$

d. $C(15,1)C(12,2) + C(15,2) C(12,1)$

e. $1 - P(15,3) P(12,3)$

f. $1 - C(15,3) C(12,3)$

11. How many three-digit even numbers can be formed using digits from the set $\{1, 2, 3, 4, 5, 6, 7\}$
- If no digit can be repeated in any one number?
 - If digits can be repeated?
12. An urn contains 5 purple, 6 orange, and 3 black marbles. In how many ways can 3 marbles be selected, without replacement, so that at least one of the three is black?
13. A group of people composed of 6 men and 3 women is to randomly select a committee of 3 people. Find the probability that all 3 are men.
14. At Dee Dee's Dinette the probability that a customer orders coffee is 0.60, the probability that a customer orders pie is 0.35, and the probability that a customer orders neither coffee nor pie is 0.30. Find the probability that a customer orders both coffee and pie.
15. A student guesses on all 8 question of a multiple choice quiz, where there are 4 answer choices for each question. Find the probability that he will get exactly 3 questions correct.
16. For the same student and quiz as in #15 above, find the probability that he will get at least one question correct.
17. Given the tree diagram with probabilities, find $\Pr[x]$. Give your answer as a decimal number with two decimal places.



18. A given school's population is 40 % women, and 60 % men. Of the school's women, 30 % are involved in music. Of the school's men, 80 % are involved in music. One student is randomly selected. Given that the selected student is involved in music, find the probability that the student is a woman. Give answer as a decimal number with three correct decimal places.

19. Given the following values of a random variable X , find $E[X]$ and the standard deviation of X .

Outcomes	Random Variable	Probability
A	1	0.15
B	2	0.20
C	3	0.30
D	5	0.35

20. A pocket contains 3 nickels, 2 dimes, and 1 quarter. Two coins are selected simultaneously and at random. A random variable, X , is defined to be the total value in cents of the two coins. Find the expected value, $E[X]$.

21. A piggy bank containing 500 unfair coins is dumped onto a table. The probability of tails is 0.8 on every coin. Let the random variable X be the number of tails that appear. Find the expected value of X .

22. A piggy bank containing 400 unfair coins is dumped onto a table. The probability of tails is 0.75 on every coin. Let the random variable X be the number of tails that appear. Find the standard deviation of X . Express your answer as a decimal number with two decimal places.

23. Let X be a normal random variable with mean 410 and standard deviation 20. Find $\Pr[360 \leq X \leq 420]$. Give four decimal places in your answer.

24. Assume that the length of angelfish is a normal random variable with mean length of 10.2 cm and standard deviation of 2.1 cm. Find the probability that a randomly selected angelfish will be at least 7 cm in length.

25. A fair coin is flipped 100 times. Let X be the number of heads on the 100 flips. Use the normal approximation to the binomial to approximate $\Pr [46 \leq X \leq 56]$.

26. Let A be a 5×7 matrix, let B be a 7×7 matrix, and let C be a 7×5 matrix. Which of the following is NOT defined?

- (a) None of the others
- (b) BC
- (c) AC
- (d) BA
- (e) CA
- (f) AB

27. Given matrices A and B , find the product AB if it is defined.

$$A = \begin{pmatrix} 3 & -7 \\ 7 & -4 \end{pmatrix} \quad B = \begin{pmatrix} -1 & -2 & 1 \\ -6 & -5 & 2 \end{pmatrix}$$

28. Given matrices C and D , find $2C - D$.

$$C = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 0 \\ -1 & 2 \end{bmatrix}$$

29. Solve using the All-Integer Method. Show your work.

$$\begin{aligned} x - 2y + 3z &= 4 \\ 4x + y + z &= 1 \\ 2x - y + z &= -1 \end{aligned}$$

30. Kylie shops exclusively at Circle Center and Keystone. If she shops at Keystone, the probability that she will shop next time at Circle Center is 0.6. If she shops at Circle Center, the probability that she will shop at Keystone next time is 0.5. If she last shopped at Keystone, what is the probability that she will shop at Keystone again in her second upcoming shopping trip (not her next trip but the one after that)?

31. Draw the transition diagram for each of the following transition matrices and indicate if it is irreducible or not and if it is regular or not.

a. $\begin{bmatrix} .2 & 0 & .8 \\ .5 & 0 & .5 \\ .1 & .7 & .2 \end{bmatrix}$

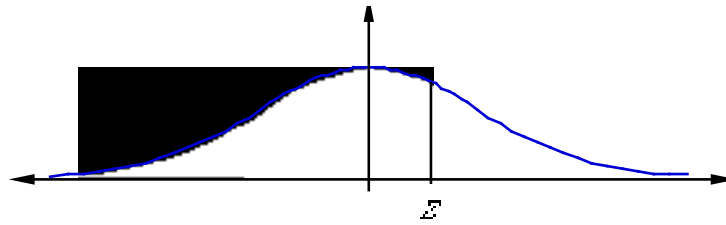
b. $\begin{bmatrix} .9 & 0 & .1 \\ .2 & 3 & .5 \\ .4 & 0 & .6 \end{bmatrix}$

32. A survey of weather conditions at a certain vacation resort indicated that if the weather on a particular day was:
Sunny (state 1), the probability was 0.6 that the next day would be sunny, 0.2 that the next day would be overcast, 0.2 that the next day would be rainy;
Overcast (state 2), the probability was 0.6 that the next day would be sunny, 0 that the next day would be overcast, 0.4 that the next day would be rainy;
Rainy (state 3), the probability was 0.4 that the next day would be sunny, 0.1 that the next day would be overcast, 0.5 that the next day would be rainy.

- a. Draw a transition diagram AND a transition matrix for the Markov chain.
- b. If it is overcast today at the resort, what is the probability that it will be sunny two days from now (not tomorrow, but the day after tomorrow).
- c. Given an initial state vector of $[0.5 \ 0.3 \ 0.2]$ for states 1, 2 and 3 in that order, find the state vector after one transition.

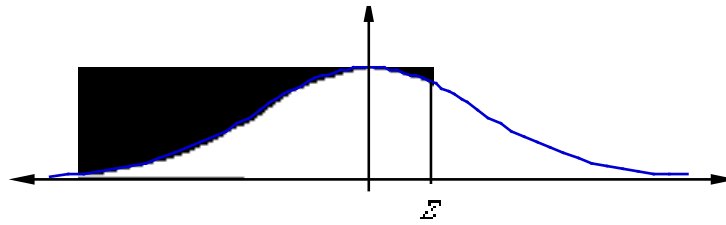
33. A Markov chain has the transition matrix $\begin{bmatrix} .3 & .7 \\ .4 & .6 \end{bmatrix}$.

- a. If the chain begins in state 2, what is the probability that it will be in state 1 after two transitions?
- b. Find the steady state vector for this Markov chain.



The table entry is the probability of being at or below the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



The table entry is the probability of being at or below the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9992	.9993