Ordinary Differential Equations
MATH 266; Class 24841
Quiz 1; August 26, 2013
Show all work to receive full/partial credit

1. (5 points) Consider the population model

\[
\frac{dP}{dt} = 0.3 \left( 1 - \frac{P}{20} \right) \left( \frac{P}{5} - 2 \right) P
\]

where \( P(t) \) is the population at time \( t \). \( A. \) For what values of \( P \) is the population in equilibrium? \( B. \) For what values of \( P \) is it increasing? \( C. \) For what values is it decreasing?

\[
\begin{array}{c|c|c|c|c|c}
\text{Interval} & 0 & \frac{P}{5} - 2 & P & \frac{dP}{dt} & \text{Behavior} \\
\hline
P < 0 & - & - & - & + & \text{increasing} \\
0 < P < 10 & - & - & + & - & \text{decreasing} \\
10 < P < 20 & + & + & - & - & \text{increasing} \\
P > 20 & + & - & - & - & \text{decreasing}
\end{array}
\]

2. (5 points) Solve the given initial value problem

\[
\frac{dy}{dt} = \frac{t}{y - t^2}, \quad y(0) = -3
\]

\[
\int y \, dy = \int \frac{t}{1-t^2} \, dt
\]

\[
u = 1-t^2, \quad du = -2t \, dt, \quad t \, dt = -\frac{du}{2}
\]

\[
y^2 = -\ln(1-t^2) + C
\]

\[
y(0) = 0 \Rightarrow \sqrt{C_1} = 0 \Rightarrow C_1 = 0
\]

\[
y(t) = -\sqrt{9 - \ln(1-t^2)}
\]