Qualitative Theory of Differential Equations
MATH 522; Class 26372
Quiz 1; August 26 2013
Show all work to receive full/partial credit

(10 points) Graphically analyze the equation
\[ \frac{dx}{dt} = 1 + 2\cos x \]
by a. sketching the vector field on the real line, b. finding all the fixed points, c. classifying their stability, and d. sketching the time-series \( x(t) \) for different initial conditions.

Then, use the linear stability analysis to support your answer by classifying equilibrium states of the system.

\[ \frac{dx}{dt} = 0 \Rightarrow 1 + 2\cos x = 0 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi k + 2\pi}{3}, \quad k = 0, 1, 2, \ldots \]

\[ x = \frac{2\pi k}{3}, \text{stable} \]

\[ x = \frac{\pi}{3}, \text{unstable} \]