1. Show by using truth tables that the statements $p \rightarrow (q \lor \neg r)$ and $\neg p \lor (q \lor \neg r)$ are equivalent. (10 points)

2. Write the negation of the statement
   
   $$(\exists x) (p(x) \land q(x)) \rightarrow r(x)$$
   
   in a way in which ‘$\neg$’ is not a main connective
   (that is, ‘$\neg$’ does not apply to a compound statement). (10 points)
3. For this problem, the universal set is the set of positive integers, \( \mathbb{N} \).

Let \( B = \{ n : 1 \leq n \leq 60 \} \).

Let \( A_2 = \{ 2k : k \in \mathbb{N} \} \), let \( A_3 = \{ 3k : k \in \mathbb{N} \} \), and let \( A_5 = \{ 5k : k \in \mathbb{N} \} \). Recalling that \( \overline{C} \) is the complement of the set \( C \), find the set

\[ B \cap \overline{A}_2 \cap \overline{A}_3 \cap \overline{A}_5 \]

4. Prove, using induction, that for all positive integers \( n \), that

\[ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3} \]
5. Define a sequence \( \{a_n\} \) by \( a_1 = 3 \) and for each positive integer \( n \), \( a_{n+1} = 2a_n + 1 \).

(a) Find the first five terms in the sequence: \( a_1, a_2, a_3, a_4, \) and \( a_5 \).

(b) Prove by induction that \( a_n > 2^n \) for every positive integer \( n \).
6. 
(a) For which integers, \( n \), is the integer \( n^2 + 4n + 3 \) divisible by 2.

(b) For which integers, \( n \), is the integer \( n^2 + 4n + 3 \) divisible by 3.
7. Let $\mathbb{N}$ be the set of positive integers and let $T = \{ m \in \mathbb{N} : m \text{ is not divisible by 3} \}$. We know the set $T$ is countable because it is a subset of the integers.

(a) Find a one-to-one correspondence between $\mathbb{N}$ and $T$ by explicitly describing a function $f : \mathbb{N} \rightarrow T$. (Hint: One way to do this is by mapping the odd integers to the numbers that have remainder 1 when divided by 3 and mapping the even integers to numbers that have remainder 2 when divided by 3.)

(b) For the function $f$ you defined in (a) above, what are $f(47)$ and $f(34)$?

(c) For the function $f$ you defined in (a) above, for which $n$ is $f(n) = 121$? for which $n$ is $f(n) = 98$?