1. Suppose \( h \) is defined on the interval \( I \) and strictly increasing on that interval. Prove that \( h \) is one-to-one on \( I \).

In the following, we will invent a new function and develop some of its properties.

Define the function \( S \) by, for \( x \) a real number,
\[
S(x) = \int_{0}^{x} \frac{dt}{\sqrt{t^2 + 1}}
\]

Using the Riemann sums for this integral (with \( n = 100 \)), it follows that \( S(1) = .881 \) to three decimal places.

2. Explain why \( S \) is defined for every real number. This shows that the domain of \( S \) is \( \mathbb{R} \).

3. Find \( S'(x) \) and \( S''(x) \) and use your results to show that \( S \) is strictly increasing on \( \mathbb{R} \).

4. Find a relationship between \( S(x) \) and \( S(-x) \).

5. Show that \( \sqrt{t^2 + 1} < t + 1 \) for \( t > 0 \) and use the inequality to show that \( S(x) > \ln(x + 1) \) for \( x > 0 \).

6. Find \( \lim_{x \to \infty} S(x) \) and \( \lim_{x \to -\infty} S(x) \). What is the range of \( S \), that is, what is the set \( \{ y : y = S(x) \text{ for some } x \in \mathbb{R} \} \)?

7. Use the results of the previous exercises to draw a graph of \( S \).

8. Use an argument similar to that of Exercise 5 above to see that there is a constant \( C \) so that \( S(x) < C + \ln(x - 1) \) for \( x \geq 2 \).

**Challenge Problem:**
(This problem will never be assigned or collected. There are solutions that are easy to understand, but there are no solutions that are easy to find!)

Find a function \( f \) that maps \([0, 1]\) one-to-one and onto \((0, 1)\).