Homework 10C

In the last part of this course, we will be working some with complex numbers and you should know about complex numbers as part of your general mathematical education. If you are unfamiliar with complex numbers, you will need to learn a little about them. Any source is acceptable; some possibilities are high school algebra or “senior math” texts, “engineering math” texts, complex analysis texts. For example, Churchill’s Complex Variables is on reserve in the University Library, and pages 1 – 18 cover the material on the complex numbers. The ability to solve the following problems indicates adequate knowledge for this course. These problems will not be handed in. There will be a quiz on November 20 that looks very much like these problems.

Practice Problems

1. Let \( z = 4 - 5i \).
   Find: (a) \( \text{Re}(z) \)  (b) \( \text{Im}(z) \)  (c) \(|z|\)  (d) \( \bar{z} \)

2. Compute:
   (a) \((3 + 2i)(2 - i) + i(-2 + i)\)
   (b) \((2 - 3i)^2(4 + 2i)\)
   (c) \((2 - i)^2 + (1 + 3i)^2\)
   (d) \(((2 - i))^2 + ((1 + 3i))^2\) see (c)
   (e) \(\frac{1}{3 + 4i}\)
   (f) \(\frac{4 - 2i}{1 + 7}\)
   (g) \(\frac{2 + 3i}{(2 - i)^2} + \frac{i}{1 + 1}\)
   (h) \(|\frac{1 + 3i}{(2 - i)}|\)

3. Find all (3) roots of the equation
   \[ z^3 - 3z^2 + 7z - 5 = 0 \]

Answers:

1. (a) 4 (b) -5 (not -5i!) (c) \(\sqrt{41}\) (d) 4 + 5i.

2. (a) 7 - i (b) 4 - 58i (c) -5 + 2i (d) -5 - 2i [i.e. the conjugate of (c)]
   (e) \(\frac{3}{25} - \frac{4}{25}i\) (f) 1 - 3i (g) 0.26 + 1.18i (h) \(\frac{|1 + 3i|}{|2 - i|} = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}\)

3. The roots are 1, 1 + 2i, and 1 - 2i.