

Susan Williams

Knots, algebraic dynamics and Mahler measure

Abstract: We will give an overview of joint work with Daniel Silver on applications of algebraic dynamics to the theory of knot and links. We study a classic link invariant, the Alexander module, via its Pontryagin dual, a compact abelian group with a \mathbf{Z}^d -action by automorphisms, where d is the number of components of the link. We use a theorem of Lind, Schmidt and Ward on growth of periodic points to give a topological interpretation of the Mahler measure of the Alexander polynomial.

The Mahler measure of a polynomial of d variables is its geometric mean over the multiplicative d -torus obtained by restricting each variable to the unit circle. When $d = 1$, this is simply the absolute value of the leading coefficient times the product of the moduli of the roots outside the unit circle. D. H. Lehmer asked in 1933 if a polynomial in $\mathbf{Z}[t]$ can have Mahler measure arbitrarily close to, but greater than, 1. The nearest known value occurs as the Mahler measure of the Alexander polynomial of a knot. Examples of links with Alexander polynomials of small Mahler measure exhibit intriguing geometric properties.