

Consider the fixed rate mortgage of  $M$  dollars, for  $T$  months, with yearly interest  $p$  %. We want to calculate the monthly payment  $x$  (in dollars); without the escrow. Denote the debt after  $t$  months by  $d(t)$ . Then,

$$d(0) = M, \quad d(T) = 0.$$

Each month we pay  $x$  dollars, which are divided between the interest and principal. The interest is  $\frac{p}{12}$  % of the current debt  $d(t)$ , that is,  $d(t)\frac{p}{1200}$  dollars. The rest is the principal, so next month the debt is

$$d(t+1) = d(t) - \left(x - \frac{d(t)p}{1200}\right) = d(t) \left(1 + \frac{p}{1200}\right) - x.$$

This means that we iterate the map  $f$ , given by

$$f(y) = \left(1 + \frac{p}{1200}\right)y - x.$$

To find the formula for the  $n$ -th iterate of  $f$ , we find the fixed point  $y_0$  of  $f$  by solving the equation

$$y_0 = \left(1 + \frac{p}{1200}\right)y_0 - x.$$

The solution is  $y_0 = 1200x/p$ . Thus, we write our map in the new coordinate  $z = y - y_0$  as

$$g(z) = \left(1 + \frac{p}{1200}\right)z,$$

so

$$g^n(z) = \left(1 + \frac{p}{1200}\right)^n z.$$

Going back to the old coordinate, we get

$$f^n(y) = \left(1 + \frac{p}{1200}\right)^n (y - y_0) + y_0 = \left(1 + \frac{p}{1200}\right)^n \left(y - \frac{1200x}{p}\right) + \frac{1200x}{p}.$$

Hence,

$$\left(1 + \frac{p}{1200}\right)^T \left(M - \frac{1200x}{p}\right) + \frac{1200x}{p} = 0.$$

Solving it we get

$$\left(1 + \frac{p}{1200}\right)^T M = \frac{1200x}{p} \left(\left(1 + \frac{p}{1200}\right)^T - 1\right),$$

so we get the formula for  $x$ :

$$x = \frac{\left(1 + \frac{p}{1200}\right)^T}{\left(1 + \frac{p}{1200}\right)^T - 1} \cdot \frac{p}{1200} \cdot M.$$