Math M111: Lecture Notes For Chapter 9 (Part 1)

Sections 9.1: Solving Quadratic Equations \( ax^2 + bx + c = 0 \)

**Method 1:** By Factoring (covered in chapter 6, section 6.5)

Example: \( 3x^2 + 5x + 2 = 0 \)  
Answer: \( x = -2/3 \) and \( x = -1 \)

**Method 2:** By Completing the Square

A. If one side is a perfect square:

a) \( 2x^2 = 50 \)  
b) \( (x - 3)^2 = 9 \)  
c) \( (x - 1)^2 = 6 \)  
d) \( x^2 - 6x + 9 = 9 \)  
e) \( (x + 1)^2 = -9 \)  
f) \( x^2 - 8x + 16 = 1 \)

B. If one side is not a perfect square, and \( a = 1 \):

Solve for \( x \): \( x^2 - x - 3 = 0 \)  
(notice that \( a = 1, b = -1, c = -3 \))

**step 1)** Rewrite it by isolating the constant by itself and make \( a = 1 \) if it is not

\( x^2 - x = 3 \)

**step 2)** Find: \( b \), \( b/2 \) and \( (b/2)^2 \)

\( b = -1 \), \( b/2 = -1/2 \), \( (b/2)^2 = 1/4 \)

**step 3)** Add \( (b/2)^2 \) to both sides: \( x^2 - x + 1/4 = 3 + 1/4 \)

the left side is always \( = (x + b/2)^2 \) or: \( (x - 1/2)^2 = 13/4 \)

**step 4)** Take the square root of both sides and solve

\[ \left( x - \frac{1}{2} \right) = \pm \sqrt{\frac{13}{4}}, \text{ two solutions:} \]

- \( \left( x - \frac{1}{2} \right) = + \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2} \) or \( x = \frac{1}{2} + \frac{\sqrt{13}}{2} \)
- \( \left( x - \frac{1}{2} \right) = - \sqrt{\frac{13}{4}} = -\frac{\sqrt{13}}{2} \) or \( x = \frac{1}{2} - \frac{\sqrt{13}}{2} \)
C. If one side is not a perfect square, and $a \neq 1$:

a) Solve for $x$: $3x^2 + 5x + 2 = 0$

   step 1) Rewrite it by isolating the constant by itself and make $a = 1$ if it is not

   $$x^2 + \frac{5}{3}x = \frac{-2}{3}$$

   step 2) Find: $b$, $b/2$ and $(b/2)^2$

   $b = 5/3$, $b/2 = 5/6$, $(b/2)^2 = 25/36$

   step 3) Add $(b/2)^2$ to both sides:

   $$x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{-2}{3} + \frac{25}{36}$$

   the left side is always $(x + b/2)^2$ or:

   $$(x + \frac{5}{6})^2 = \frac{1}{36}$$

   step 4) Take the square root of both sides and solve

   $$x = -\frac{2}{3}$$ ; $x = -1$

b) Solve for $x$: $2x^2 + 3x - 1 = 0$

   step 1) Rewrite it by isolating the constant by itself and make $a = 1$ if it is not

   $$x^2 + \frac{3}{2}x = \frac{1}{2}$$

   step 2) Find: $b$, $b/2$ and $(b/2)^2$

   $b = 3/2$, $b/2 = 3/4$, $(b/2)^2 = 9/16$

   step 3) Add $(b/2)^2$ to both sides:

   $$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$

   the left side is always $(x + b/2)^2$ or:

   $$(x + \frac{3}{4})^2 = \frac{17}{16}$$

   step 4) Take the square root of both sides and solve

   $$x = \frac{-3 + \sqrt{17}}{4}$$ ; $x = \frac{-3 - \sqrt{17}}{4}$

c) Solve for $x$: $4x^2 + 5x + 5 = 0$

d) Solve for $x$: $(2x - 2a)^2 = 3b$
Sections 9.2: Quadratic Formula

We covered method 1 and 2 in section 9.1, and now method 3 by using the Quadratic Formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

a) Solve for \( x \): \( 2x^2 - 7x = -3 \)

\[ \text{step1) Rewrite it by making it } = 0 \text{ and find } a, b \text{ and } c: \]
\[ 2x^2 - 7x + 3 = 0 \]
\[ a = 2, \quad b = -7, \quad c = -3 \]

\[ \text{step2) Find the discriminate: } \quad b^2 - 4ac = (-7)^2 - 4(2)(3) = 25 \]

\[ \text{step3) The solution is: } \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-(-7) \pm \sqrt{25}}{2(2)} \quad ; \quad x = 1/2, \quad x = 3 \]

Notice: Using the Quadratic Formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \); if the discriminate:

- \( b^2 - 4ac > 0 \) \( \Rightarrow \) there are 2 real solution
  \[ x^2 - 13x + 30 = 0; \quad b^2 - 4ac = 49; \quad x = 3 \& x = 10 \]

- \( b^2 - 4ac < 0 \) \( \Rightarrow \) there are 2 non real solution
  \[ x^2 - 4x + 13 = 0; \quad b^2 - 4ac = -24; \quad x = \pm i\sqrt{6} \]

- \( b^2 - 4ac = 0 \) \( \Rightarrow \) there is 1 real solution
  \[ x^2 - 8x + 16 = 0; \quad b^2 - 4ac = 0; \quad x = 4 \]

b) Solve for \( x \): \( 6x^2 - 1 = x \)

c) Solve for \( x \): \( x^2 - 8x = -10 \) (Not a perfect square, 2 real solutions)

d) Solve for \( x \): \( 3x^2 - 6x = -3 \) (1 real solutions)

e) Solve for \( x \): \( 3x^2 + 3 = 4x \) (2 non-real solutions)

More examples:

f) Solve for \( x \): \( x^2 + \frac{x}{3} = \frac{1}{6} \)

g) Solve for \( x \): \( (x + 1)(x - 7) = 1 \)

h) Solve for \( k \): \( k = \frac{k + 15}{3(k - 1)} \)

i) Solve for \( x \): \( (x + 5)(x - 6) = (2x - 1)(x - 4) \)

j) Solve for \( x \): \( x^2 + 4x + 11 = 0 \)
Sections 9.3: Equations in Quadratic Form

A. Solve the following (from page 544 in the book):

18) \( \frac{2}{k} + \frac{3}{k^2} = \frac{3}{2} \)

26) \( \frac{2}{3x + 2} = \frac{15}{(3x + 2)^2} \)

28) \( \frac{k}{2 - k} + \frac{2}{k} = 5 \)

B. Solve the following equations:

a) \( x^4 - 3x^2 + 2 = 0 \)

b) \( x + \sqrt{x} - 12 = 0 \)

c) \( 4x^{-2} - x^{-1} - 5 = 0 \)

d) \( m^{\frac{1}{3}} - m^{\frac{1}{6}} - 6 = 0 \)

e) \( 2 \left( \frac{1}{x} + 1 \right)^2 - 3 \left( \frac{1}{x} + 1 \right) - 20 = 0 \)

C. Story Problems

a) On a sales trip, Gail drives the 600 mi to Richmond at a certain speed. The return trip is made at a speed that is 10 mph slower. Total time for the round trip was 22 hr. How fast did Gail travel on each part of the trip?

b) The Hudson River flows at a rate of 3 mph. A patrol boat travels 60 miles upriver and returns in a total time of 9 hr. What is the speed of the boat in still water?

c) During the first part of a canoe trip, Tim covered 60 km at a certain speed. He then traveled 24 km at a speed that was 4 km/h slower. If the total time for the trip was 8 hr, what was the speed on each part of the trip?

Sections 9.4: Formulas and Applications

A. Solve the following equations:

a) \( A = \frac{kxy}{t^2 + 1} \) for \( t \)

b) \( d = vt + gt^2 \) for \( t \)

c) \( w = \sqrt{\frac{1}{Lc}} \) for \( L \)

B. Story Problems

a) The hypotenuse of a right triangle is 25 m long. The length of one leg is 17 m less than the other. Find the lengths of the legs.

b) The outside of a picture frame measures 14 in. by 20 in.; 160 in\(^2\) of picture shows. Find the width of the frame.