Math M111: Lecture Notes For Chapter 4

Sections 4.1: System of 2 equations, 2 unknowns

- **Method 1**: Solving Graphically

  Example 1: \(3x + y = 5\) and \(x - 2y = 4\)

  Example 2: \(x + 2y = -3\) and \(y - x = 6\)

  **Special Cases:**
  
  **Case 1**: If the two lines are parallel (*do not intersect*), then there is no solution. or Inconsistent
  
  \(y - x = 5\) and \(2x - 2y = 10\)

  **Case 2**: If the graph of the two equations gives only one line, then there is a Dependent solution.

  \(y = 3 - x\) and \(2x + 2y = 6\)

- **Method 2**: Solving by Elimination

  Example 3: \(5x - 7y = -16\) and \(2x + 8y = 26\)

  The following examples from the book:

  22) \(8x + 4y = 0\) and \(4x - 2y = 2\)
  24) \(x - 4y = 2\) and \(4x - 16y = 8\)
  26) \(2x - 3y = 7\) and \(-4x + 6y = 14\)
  32) \(\frac{1}{5}x + y = \frac{6}{5}\) and \(\frac{1}{10}x + \frac{1}{3}y = \frac{5}{6}\)

- **Method 3**: Solving by Substitution

  Example 4: \(4x + y = -1\) and \(x - 2y = 11\)

  The following examples from the book:

  46) \(6x - y = -9\) and \(4 + 7x = -y\)
  52) \(x = 3y\) and \(3x - 9y = 0\)
  54) \(y = -4x\) and \(8x + 2y = 4\)
Section 4.2: System of 3 equations

Solve the following system of equations

**Example 1**
\[
\begin{align*}
2x + y - 3z &= -4 \\
4x - 2y + z &= 9 \\
3x + 5y - 2z &= 5
\end{align*}
\]

**Example 2**
\[
\begin{align*}
3x + 2z &= 11 \\
y - 7z &= 4 \\
x - 6y &= 1
\end{align*}
\]

30) 
\[
\begin{align*}
-2x + 5y + z &= -3 \\
5x + 14y - z &= -11 \\
7x + 9y - 2z &= -5
\end{align*}
\]

32) 
\[
\begin{align*}
x + 4y - z &= 3 \\
-2x - 8y + 2z &= -6 \\
-3x + 12y - 3z &= 9
\end{align*}
\]

Section 4.2: Applications

14. If $5000 is invested in an account paying simple annual interest, how much interest will be earned during the first year at the following rates? (a) 2% (b) 3% (c) 4% (d) 3.5%

18. How many liters each of 15% acid and 33% acid should be mixed to get 120 L of 21% acid?

20. A truck radiator holds 36 L of fluid. How much pure antifreeze must be added to a mixture that is 4% antifreeze to fill the radiator with a mixture that is 20% antifreeze?

22. A popular fruit drink is made by mixing fruit juices. Such a drink with 50% juice is to be mixed with another drink that is 30% juice to get 200 L of a drink that is 45% juice. How much of each should be used?

24. An investor will invest a total of $15,000 in two accounts, one paying 4% annual simple interest, and the other 3%. If he wants to earn $550 annual interest, how much should he invest at each rate?

28. A freight train and an express train leave towns 390 km apart, traveling toward one another. The freight train travels 30 km per hr slower than the express train. They pass one another 3 hr later. What are their speeds?

30. Traveling for 3 hr into a steady headwind, a plane flies 1650 mi. The pilot determines that flying with the same wind for 2 hr, he could make a trip of 1300 mi. Find the speed of the plane and the speed of the wind.

32. Carol Britz plans to mix pecan clusters that sell for $3.60 per lb with chocolate truffles that sell for $7.20 per lb to get a mixture that she can sell in Valentine boxes for $4.95 per lb. How much of the $3.60 clusters and the $7.20 truffles should she use to create 80 lb of the mix?

34. At a business meeting at Panera Bread, the bill for two cappuccinos and three house lattes was $10.95. At another table, the bill for one cappuccino and two house lattes was $6.65. How much did each type of beverage cost?