Math M111: Lecture Notes For Chapter 7

Sections 7.1: Multiplying and Dividing Rational Expressions

Find the domain of (page 387):

a) \( \frac{x + 3}{x} \)  
b) \( \frac{x - 3}{x - 8} \)  
c) \( \frac{x^2 - 3x - 4}{x^2 - 18x + 77} \)
d) \( \frac{5x - 10}{3} \)  
e) \( \frac{3x - 2}{x^2 + 1} \)  

16) \( \frac{2x + 4}{3x^2 + 11x - 42} \)

Write each rational expression in the lowest term (page 388):

a) \( \frac{12a - 6}{6} \)  
b) \( \frac{x^2 - y^2}{x - y} \)  
c) \( \frac{x^2 + 2x - 3}{x + 3} \)
d) \( \frac{x^2 - 25}{x^2 + 7x + 10} \)  

42) \( \frac{12x^2 - 4x - 5}{8x^2 - 6x - 5} \)  

44) \( \frac{x^3 - y^3}{x - y} \)  

46) \( \frac{3x^2 - 9xy - 54y^2}{3x^2 - 6xy - 72y^2} \)  

48) \( \frac{2xy + 2xw + y + w}{2xy + y - 2xw - w} \)  

54) \( \frac{(8 - p)(x + 2)}{(p - 8)(x - 2)} \)

56) \( \frac{7x - 21}{63 - 21x} \)

Multiply and divide as indicated (page 388):

a) \( \frac{x^2 - 9}{x + 4} \cdot \frac{x + 4}{x - 3} \)  
b) \( \frac{2x - 10}{x - 4} \cdot \frac{x^2 - 8x + 16}{x - 5} \)

c) \( \frac{x^2 - 4}{x^2 + 7x + 12} \cdot \frac{x^2 + 6x + 9}{x^2 + x - 6} \)  

74) \( \frac{a^2 - 1}{4a} \cdot \frac{2}{1 - a} \)

d) \( \frac{x^2 - 1}{x^2 - 2x + 1} \div \frac{x + 1}{x - 1} \)  

78) \( (y^2 - 4) \div \frac{2 - y}{8y} \)

86) \( \frac{8a^2 - 6ab - 9b^2}{6a^2 - 5ab - 6b^2} \div \frac{4a^2 + 11ab + 6b^2}{9a^2 + 12ab + 4b^2} \)

88) \( \frac{16c^2 + 24cd + 9d^2}{16c^2 - 16cd + 3d^2} \div \frac{16c^2 - 9d^2}{16c^2 - 24cd + 9d^2} \)
Sections 7.2: Adding and Subtracting Rational Expressions

Find the LCD of:

a) \( x^2 - y^2 \) and \( 4x - 4y \)

b) \( x^2 - 4 \); \( x^3 + x^2 - 6x \) and \( 3x + 6 \)

c) \( x^2 + 10x + 25 \) and \( x^2 + 2x - 15 \)

d) \( y^2 - 9 \) and \( 3 - y \)

Add or Subtract as indicated (page 396):

\[
\begin{align*}
\text{a) } & \quad \frac{5}{x^2 + 2x - 8} + \frac{3}{2 - x} \\
\text{b) } & \quad \frac{x - 1}{x - 2} - \frac{x + 1}{x + 2} + \frac{x - 6}{x^2 - 4} \\
\text{c) } & \quad \frac{8a}{a^2 - 1} + \frac{2}{1 - a} - \frac{4}{a + 1} \\
\text{d) } & \quad \frac{4}{x + 1} - \frac{6x - 2}{x^2 - 1} - \frac{3}{1 - x} \\
\text{68) } & \quad \frac{8}{(3x - 1)^2} + \frac{2}{(3x - 1)^2} - 6 \\
\text{78) } & \quad \frac{5}{x^2 + 6x + 9} - \frac{2}{x^2 + 4x + 3}
\end{align*}
\]

Sections 7.3: Complex Fraction

Simplify each complex fraction (page 404):

\[
\begin{align*}
\text{a) } & \quad \frac{1 - \frac{x}{y - x}}{1 - \frac{3x}{y + x}} \\
\text{b) } & \quad \frac{5 - \frac{6}{x - 5}}{\frac{5 - x}{x - 5}} \\
\text{18) } & \quad \frac{p - \frac{p + 2}{4}}{\frac{3 - \frac{5}{4}}{2p}} \\
\text{20) } & \quad \frac{\frac{y + 3}{y - 1}}{\frac{\frac{4}{y}}{y}} \\
\text{c) } & \quad \frac{1 + \frac{2x^{-1}}{1 - 3x^{-1}}}{\frac{2}{y^{-1} + x^{-1}}/x^2 - 4y^{-2}} \\
\text{d) } & \quad \frac{\frac{2y^{-1} + x^{-1}}{x^2 - 4y^{-2}}}{\frac{a^2 - 4b^{-2}}{3b - 6a}} \\
\text{e) } & \quad \frac{a^2b^{-2} - b^2a^{-2}}{ab^{-1} - ba^{-1}}
\end{align*}
\]
Sections 7.4: Equations with Rational Expressions

Solve each equation (page 410):

a) \[ \frac{5}{x+4} = \frac{3}{x-2} \]

b) \[ \frac{x+4}{x^2+5x+6} = 0 \]

c) \[ \frac{3}{x^2-2x} + \frac{2x-1}{x^2+2x-8} = \frac{2}{x+4} \]

d) \[ \frac{2}{x^2-1} - \frac{2}{x+1} = \frac{-1}{1-x} \]

e) \[ \frac{3x}{x^2+2x-8} - \frac{3}{2x-x^2} = \frac{2}{x+4} \]

Graph the following rational function. Give the equation of the vertical asymptote: \[ f(x) = \frac{2}{x-2} \]

Sections 7.5: Applications of Rational Expressions

Solve each formula for the specified variable (page 423):

a) \[ \frac{1}{x} + \frac{2}{y} = \frac{3}{z} \] for \( x \)

b) \[ x = \frac{2b}{b-3y} \] for \( b \)

c) \[ \frac{a}{b} = \frac{2x+y}{x} \] for \( x \)

d) \[ y = \frac{ab}{2b+3a} \] for \( a \)

e) To determine the number of deer in a game preserve, a conservationist catches 318 deer, tags them, and lets them loose. Later, 168 deer are caught; 56 of them are tagged. How many deer are in the preserve?

24) In a certain southern state, sales tax on a purchase of $1.50 is $0.12. What is the sales tax on a purchase of $6.00?

f) The speed of the current in Catamount Creek is 3 mph. Paul’s kayak can travel 4 mi upstream in the same time that it takes to travel 10 mi downstream. What is the speed of Paul’s kayak in still water?

g) A truck driver drives every week to a city in the east cost, if he drives 45 Mph he would arrive there 1 hour early. If he drives 40 Mph, he would arrive 1 hour late. How far is the city?

44) Johnny averages 30 mph when he drives on the old highway to his favorite fishing hole, and he averages 50 mph when most of his route is on the interstate. If both routes are the same length, and he saves 2 hr by traveling on the interstate, how far away is the fishing hole?

46) While on vacation, Jim and Annie decided to drive all day. During the first part of their trip on the highway, they averaged 60 mph. When they got to Houston, traffic caused them to average only 30 mph. The distance they drove in Houston was 100 mi less than their distance on the highway. What was their total driving distance if they spent 50 min more on the highway than they did in Houston?

h) Alexander can paint a room in 4 hr. Alexandra can paint the same room in 3 hr. Working together, how long will it take them to paint the room?

52) A winery has a vat to hold chardonnay. An inlet pipe can fill the vat in 9 hr, while an outlet pipe can empty it in 12 hr. How long will it take to fill the vat if both the outlet and the inlet pipes are open?