Math 153: Lecture Notes For Chapter 3

Section 3.1: Rectangular Coordinate System

• **Note 1:** The rectangular coordinate system is divide into 4 quadrants: I, II, III and IV.

  
  ![Coordinate System]

  - I: +, +
  - II: -, +
  - III: -, -
  - IV: +, -

• **Note 2:** Distance between two points \(A(x_1,y_1)\) and \(B(x_2,y_2)\) is: 
  \[D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

  **Example 1:** Find the distance between \(A(1, 2)\) and \(B(-1, 4)\)

• **Note 3:** Midpoint of a line connecting 2 points \(A(x_1,y_1)\) and \(B(x_2,y_2)\):
  \[
  \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
  \]

  **Example 2:** Find the midpoint of from \(A(-1, 4)\) and \(B(1, 2)\)

  **Example 3:** Find a formula that expresses the fact that an arbitrary point \(P(x, y)\) is on the perpendicular bisector \(l\) of segment \(AB\):

  \[A(-3, 2), B(5, -4)\]

More examples:

  **Example 4:** Find all points on the y-axis that are a distance of 4 from \(P(3,5)\)

  **Example 5:** Find all points on the x-axis that are a distance of 5 from \(P(-2,3)\)

Section 3.2: Graphs of Equations

1) graph \[y = -x^2 + 5\]
2) graph \[y = \sqrt{x - 4}\]

• Symmetric with respect to the y-axis, then \(f(x) = f(-x)\) such as: \(y = x^2 + 2\)

• Symmetric with respect to the x-axis, then \(f(y) = f(-y)\) such as: \(x = y^2 + 3\).

• Symmetric with respect to the origin, then simultaneous substitution of \(-x\) for \(x\) and \(-y\) for \(y\) gets the same equation such as: \(2y = x^3\)

**Examples:** determine which graphs are symmetric with respect of:

- **a) the y-axis**
  3) \(y = \frac{1}{2}x^2\)
  6) \(x = 2y^2 - 4\)

- **b) the x-axis**
  4) \(y = -x^2 + 2\)
  7) \(y = 3x^3\)

- **c) the origin**
  5) \(x = -2y^2\)
Equation Of The Circle With a Radius \( r \):

\[
(x - h)^2 + (y - k)^2 = r^2
\]

8) Find the equation of the circle with center (-4, 1) and \( r = 3 \)

9) Find the equation of the circle with center (-2, 4) and passes (2, 1)

10) Find the equation of the circle with center (4, -1) and tangent to the \( x \)-axis.

11) Find the equation of the circle with end points of a diameter \( A(-5, 2) \) and \( B(3,6) \)

12) Find the center and the radius of \( x^2 + y^2 + 8x - 10y + 37 = 0 \)

13) Find the center and the radius of \( 9x^2 + 9y^2 + 12x - 6y + 4 = 0 \)

14) Find the center and the radius of \( x^2 + y^2 + 4x + 6y + 16 = 0 \)

15) Graph the circle or semicircle: \( x^2 + y^2 = 8 \)

16) Graph the circle or semicircle: \( (x - 4)^2 + (y + 2)^2 = 4 \)

17) Graph the circle or semicircle: \( x^2 + y^2 = 16 \)

18) Graph the circle or semicircle: \( y = \sqrt{16 - x^2} \) (half the circle of example 17 when \( y \) is isolated)

19) Graph the circle or semicircle: \( x = -\sqrt{16 - y^2} \) (half the circle of example 17 when \( x \) is isolated)

Important:
- If \( y = +\sqrt{\ldots} \), then the graph is the upper half of a circle (see example 18).
- If \( y = -\sqrt{\ldots} \), then the graph is the lower half of a circle.
- If \( x = -\sqrt{\ldots} \), then the graph is the left half of a circle (see example 19)
- If \( x = +\sqrt{\ldots} \), then the graph is the right half of a circle

Book, Exer. 57 - 60: Find the equations for the upper half, lower half, right half, and left half of the circle:

58) \( (x + 3)^2 + y^2 = 64 \) \hspace{1cm} 60) \( (x - 3)^2 + (y - 5)^2 = 4 \)

Book, Exer. 61 - 62: Determine whether the point \( P \) is inside, outside, or on the circle with center \( C \) and radius \( r \):

62) (a) \( P(3,8), C(-2,-4), r = 13 \) \hspace{1cm} (b) \( P(-2,5), C(3,7), r = 6 \)

Book, Exer. 63 - 64: For the given circle, find (a) the \( x \)-intercepts and (b) the \( y \)-intercepts:

64) \( x^2 + y^2 - 10x + 4y + 13 = 0 \)
Section 3.3: Lines

- **Slope of a line** = \( \frac{\text{Vertical Changes}}{\text{Horizontal Changes}} = \frac{\text{Diff. in } y}{\text{Diff. in } x} = \frac{y_2 - y_1}{x_2 - x_1} \)

1) Find the slope of the line passing the points (-2, 4) and (3, -1)

2) Find the slope of the line passing the points (2,-3) and (-1, 3)

- \( m > 0 \) or positive slope, then the line is increasing or rising

- \( m < 0 \) or negative slope, then the line is decreasing or falling

- \( m = 0 \), then the line is horizontal

- \( m = \text{undefined} \), no slope, then the line is vertical

3) graph the line that passes the point \( P(2, 4) \) and has \( m = -1/3 \)

- **Slope-Intercept Equation:**

  \[ y = mx + b \quad (m \text{ is the slope, } b \text{ is the y-intercept}) \]

Find the slope and the y-intercept for:

4) \( 4x + 2y = 5 \)

5) \( 3y - 2x = 5 + 9y - 2x \)

6) \( 5x = 2/3 \ y - 10 \)

7) \( y = 10 \)

8) \( x = 5 \)

- **General Linear Equation:**

  \[ ax + by = c \]

Note: isolate \( y \) in the above equation and you will get:

\[ y = \frac{-a}{b} x + \frac{c}{b} \]

Compare it to the Slope-Intercept Equation: slope = \(-a/b\) and the y-intercept = \(c/b\).
Graphing a Linear Equation:

- **Using the x and y-intercepts** (x-intercepts when y = 0; y-intercepts when x = 0)
  
  9) \(4x - 5y = 20\) 
  10) \(3x + 2y = 12\) 
  11) \(2y - 3x = 0\)

- **Using Slope-Intercept Equation** (isolate y to get \(y = mx + b\))
  
  12) \(2y - 3x = 0\) 
  13) \(-2x + y = 3\)

- **Special Cases:**
  
  14) \(y = 3\) 
  15) \(x = 5\) 
  16) \(y = 0\)

Finding the equation of a line: \(y = mx + b\) (or \(ax + by = c\))

- **Case 1: One point is given and the slope:**

  Find the equation of the line having the given slope and containing the given point:

  17) \(m = -2\); (2, 8) 
  18) \(m = -4/5\); (2, 3)

- **Case 2: Two points are given:**

  Find the equation of the line containing the given pair of points:

  19) (2, -1) and (1, -3) 
  20) (-1, -5) and (2, 1)

Parallel and perpendicular lines:

- Two lines are **parallel** if they have same slope but different y-intercepts.
  
  Example: \(y = 3x - 4\) and \(y = 3x - 2\) where the slope is 3 in both

- Two line are **perpendicular** if the slopes are \(m\) and \(-1/m\) (multiplication of both slopes = -1)
  
  Example: \(y = 2x - 5\) and \(y = -1/2x + 3\) where \(m = 2\) and \(m = -1/2\)

  Find an equation of the line containing the given point and **parallel** to the given line:

  21) \((2, -1)\); \(2y + 10 = x\) 
  22) \((-8, 4)\); \(2y - 2x = -17\)

  Find an equation of the line containing the given point and **perpendicular** to the given line:

  23) \((-3, 4)\); \(y - 3x = -2\) 
  24) \((2, -3)\); \(2y + 4x = 1\)

  Find an equation of the perpendicular bisector of the segment \(AB\):

  25) \(A(4, 2), B(-2, 10)\)

Applications:

26) The cost of producing 100 units is $60 and for 120 units is $70. Find the equation, and the cost of 200 units