Math 153: Lecture Notes For Chapter 3 (Part b)

Section 3.4: Definition of Function

Function: is a rule that assigns to each element \( x \) (input) in a set \( D \) exactly one element, called \( f(x) \) (output) in a set \( E \).

Graph A: A function, vertical line crosses only once. Different input, Different output.

Graph B: A function, vertical line crosses only once. Different input, Same output (the profit in two different years were the same).

Graph C: Not a function, vertical line crosses more than once. Same input, Different output (two different profits for the same year).

Vertical Line Test: If any vertical line meets the graph of an equation only once, then the equation is a function.

Examples:

1) If \( y = x^2 \) Find the value of \( y \) if \( x = -1, x = 1 \), is this equation a function?

2) If \( x = y^2 \) Find the value of \( x \) if \( y = -1, y = 1 \), is this equation a function?

3) If \( f(x) = -x^2 + 2x + 2 \), find:
   a) \( f(0) \)  
   b) \( f(-1) \)  
   c) \( f(a+b) \)  
   d) \( f(a) + f(b) \)

4) If \( f(x) = 2x^2 + 3x - 1 \), find:
   a) \( f(a) \)  
   b) \( f(-a) \)  
   c) \( f(a+h) \)  
   d) \( \frac{f(a+h) - f(a)}{h} \)
**Domain:** The set of all possible inputs that would not cause:

a) a zero in the denominator
b) negative under an even root

Examples: Find the domain of the following:

5) \( f(x) = \frac{x+5}{x-4} \)
6) \( f(x) = \sqrt{5-x} \)
7) \( f(x) = \frac{x+5}{\sqrt{5-x}} \)
8) \( f(x) = \frac{5}{(x-2)\sqrt{x+2}} \)

9) Find the function \( f(x) \) if it is linear and: \( f(-1) = 2, f(2) = 3 \).

For the following examples,

a) Sketch the function

   a) Find the range and the domain.

   c) The intervals in which the function is increasing, decreasing or constant.

10) \( f(x) = x^2 - 1 \)
11) for \( f(x) = \sqrt{4-x^2} \)
12) for \( f(x) = \sqrt{4+x} \)
13) for \( f(x) = -3x + 2 \)
Section 3.5: Graphs of Functions

**Even Function:**
\[ f(x) = f(-x) \]
Symmetric with respect to \( y \) axis

- Example:
  \[ f(x) = x^2 + 2 \]
  \[ f(-x) = (-x)^2 + 2 = x^2 + 2 = f(x) \]

**Odd Functions:**
\[ f(-x) = -f(x) \]
Symmetric with respect to origin

- Example:
  \[ f(x) = 3x^3 + 2x \]
  \[ f(-x) = 3(-x)^3 + 2(-x) = -3x^3 - 2x = -f(x) \]

**Reflecting Graphs in the \( x \)-axis:**
\[ y = f(x) \text{ and } y = -f(x) \]
Reflect in the \( x \)-axis

- Example: \( y = x^2 \text{ and } y = -x^2 \)

**Reflecting Graphs in the \( y \)-axis:**
\[ y = f(x) \text{ and } y = f(-x) \]
Reflect in the \( y \)-axis

- Example: \( y = \sqrt{x} \text{ and } y = \sqrt{-x} \)
Vertically Shifting the Graph Upward:

\[ y = f(x) + c \]

Shift graph upward \( c \) units. \((c > 0)\)

Example: \( f(x) = x^2 + 1 \)

and \( f(x) = x^2 + 5 \)

Vertically Shifting the Graph Downward:

\[ y = f(x) - c \]

Shift graph downward \( c \) units. \((c > 0)\)

Example: \( f(x) = x^2 - 1 \)

and \( f(x) = x^2 - 5 \)

Horizontally Shifting the Graph to the Right:

\[ y = f(x - c) \]

Shift graph to the right \( c \) units. \((c > 0)\)

Example: \( f(x) = (x - 1)^2 \)

and \( f(x) = (x - 5)^2 \)

Horizontally Shifting the Graph to the Left:

\[ y = f(x + c) \]

Shift graph to the left \( c \) units. \((c > 0)\)

Example: \( f(x) = (x + 1)^2 \)

and \( f(x) = (x + 5)^2 \)
Vertically Stretching:
\[ y = c \cdot f(x) \]
\((c > 1)\)
Example: \( y = x^2 \) and \( y = 2x^2 \)
If \( x = -2 \), Then \( y = 4 \) is stretched to \( y = 8 \)
Vertically Stretched by a factor of 2

Vertically Compressing:
\[ y = c \cdot f(x) \]
\((0 < c < 1)\)
Example: \( y = x^2 \) and \( y = \frac{1}{2}x^2 = 0.5x^2 \)
If \( x = -2 \), Then \( y = 4 \) is compressed to \( y = 2 \)
Vertically Compressed by a factor of \( \frac{1}{0.5} = 2 \)

Horizontally Compressing:
\[ y = f(cx) \]
\((c > 1)\)
Example: \( y = x^2 + 3x + 4 \) and \( c = 2 \), then
\[ y = f(2x) = (2x)^2 + 3(2x) + 4 \]
Horizontally Compressed by a factor of 2

Horizontally Stretching:
\[ y = f(cx) \]
\((0 < c < 1)\)
Example: \( y = x^2 + 3x + 4 \) and \( c = 0.5 \), then
\[ y = f(0.5x) = (0.5x)^2 + 3(0.5x) + 4 \]
Horizontally Stretched by a factor of \( \frac{1}{0.5} = 2 \)
**Graphs with Absolute Value:**

Example: \( y = |x - 1| \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The break is where \( y = 0 \) or \( x = 1 \)

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**Graphs of Piecewise-Defined function**

\[
f(x) = \begin{cases} 
-x & \text{for } x < -1 \\
2x^2 & \text{for } -1 \leq x < 1 \\
-1 & \text{for } x \geq 1 
\end{cases}
\]

The graph of this function is divided into three parts:

<table>
<thead>
<tr>
<th>( x &lt; -1 )</th>
<th>(-1 \leq x &lt; 1)</th>
<th>( x \geq 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = -x )</td>
<td>( f(x) = 2x^2 )</td>
<td>( f(x) = -1 )</td>
</tr>
<tr>
<td>( x = 0 ) Out</td>
<td>( x = 0 ) ( f = 0 )</td>
<td>( x = 1 ) ( f = -1 )</td>
</tr>
<tr>
<td>( x = -2 ) ( f = 2 )</td>
<td>( x = -1 ) ( f = 2 )</td>
<td>( x = 3 ) ( f = -1 )</td>
</tr>
<tr>
<td>( x = -4 ) ( f = 4 )</td>
<td>( x = 1 ) ( f = 2 )</td>
<td>( \text{Horizontal line} )</td>
</tr>
</tbody>
</table>

The graphing will be done in class.

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**Graphs with Greatest Integer:** \( \|x\| = n \)

The left side is always greater or equal to the right side. An easier way to look at it:

*If are driving (in positive direction), your location is always equal to the last mile (in integer value) that you passed.*

Examples:

\[ [1.2] = 1, \quad [2.3] = 2 \]
\[ [-0.5] = -1, \quad [0.8] = 0 \]
\[ [3.6] = 3 \quad [-2.8] = -3 \quad [4.1] = 4 \]
Determine whether the functions are even, odd or neither:

1) \( f(x) = x^5 + x \)  
2) \( f(x) = 1 - x^4 \)  
3) \( f(x) = |x| - 5 \)  
4) \( f(x) = \sqrt{x^4 + 2} \)  
5) \( f(x) = \frac{2}{x} + x^3 \)

Sketch, on the same coordinate plane, the graphs of \( f \) for the given values \( c \).

6) \( f(x) = 2x^2 - c \); \( c = -4, 2, 4 \)  
7) \( f(x) = (x + c)^3 \); \( c = -1, 2, 3 \)  
8) \( f(x) = cx^3 \); \( c = -0.5, 2, 3 \)

Using the following graph of a function \( f \). Sketch the graph of the given functions

9) \( y = f(x) + 2 \)  
10) \( y = f(x - 1) \)  
11) \( y = f(x - 1) + 2 \)  
12) \( y = 2f(x) \) and \( y = -2f(x) \)  
13) \( y = f(3x) \)

\[ y \]
\[ x \]

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**Book, Exer. 25 - 30:** If the point \( P \) is on the graph of a function \( f \), find the corresponding point on the graph of the given function:

26) \( P(3, -1) \); \( y = 2f(x) + 4 \)  
28) \( P(-2, 4) \); \( y = 1/2f(x - 3) + 3 \)  
30) \( P(-2, 1) \); \( y = -3f(2x) - 1 \)

**Book, Exer. 31 - 38:** Explain how the graph of the function compares to the graph of \( y = f(x) \):

32) \( y = 3f(x - 1) \)  
34) \( y = -f(x + 4) \)  
36) \( y = f(1/2x) - 3 \)

Sketch the graph of:

14) \( f(x) = \begin{cases} 
  x + 4 & \text{for } x \leq -2 \\ 
  x^2 - 1 & \text{for } |x| < 2 \\ 
  2 & \text{for } x \geq 2 
\end{cases} \)  
15) \( f(x) = \|x + 2\| \)