Problem 1. Find $x$ and $y$ intercepts of the graph: $x^2 + 2y^2 - 6x + 2y + 8 = 0$.

Solution. First we set $y = 0$ and find $x$-intercepts:

$$x^2 - 6x + 4 = 0$$

$$x = \frac{6 \pm \sqrt{6^2 - 4 \cdot 4}}{2} = 3 \pm \sqrt{5}.$$

Then we set $x = 0$ and get

$$2y^2 + 2y + 4 = 0.$$

Since the discriminant is $2^2 - 4 \cdot 4 \cdot 2 = -28 < 0$, there is no solution and therefore no $y$-intercept.

\qed

Problem 2. What is the graph of the equation: $-2x^2 - 2y^2 + 12x - 8y - 6 = 0$?

Solution. We divide by $-2$ first and then complete the square:

$$(x^2 - 6x) + (y^2 + 4y) = -3,$$

$$(x^2 - 6x + 3^2) + (y^2 + 4y + 2^2) = -3 + 3^2 + 2^2,$$

$$(x - 3)^2 + (y + 2)^2 = 10.$$

Therefore the graph is the circle with center $(3, -2)$ and radius $\sqrt{10}$.

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