Polar Coordinates

In the Cartesian coordinate system, a point in the plane is described by an ordered-pair \((x, y)\), where \(x\) and \(y\) representing the distance from the \(y\)- and \(x\)-axis respectively to the point.

In the polar coordinate system, a point in the plane is described by an ordered-pair \((r, \theta)\), where \(r\) represents the distance from the pole (a point chosen as reference) to the point, and \(\theta\) represents the angular measure (in radians), in a counterclockwise direction, from the polar axis (a ray (half-line) emanating from the pole and traditionally chosen to align with the horizontal \(x\)-axis of the Cartesian system to facilitate comparison and conversion) to the ray connecting the pole and the point.

While an ordered-pair in either coordinate system locates a unique point in the plane, a given point in the plane has a unique ordered-pair representation only in the Cartesian coordinate system. In the polar coordinate system, the point represented by \((a, \alpha)\) can also be represented by \((a, \alpha + 2n\pi)\), \(n\) any integer. Furthermore, with the convention of interpreting \((-r, \theta)\) as \((r, \theta + \pi)\), the point \((a, \alpha)\) can be represented as \((-a, \alpha + \pi)\). Hence the representation of a given point in the plane is not unique in polar coordinates.

The Cartesian (rectangular) coordinates and polar coordinates are related by:

\[ x = r \cos \theta, \quad y = r \sin \theta \]

or, equivalently,

\[ r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1} \frac{y}{x} \]

The pole is \((0, \theta)\) for any value of \(\theta\).

Polar Curves

The graph of a polar equation \(r = f(\theta)\), or \(F(r, \theta) = 0\), is the locus of all points that have a polar representation \((r, \theta)\) that satisfies the equation. Such a graph is also known as a polar curve.

The best way to sketch a polar curve is to first sketch the Cartesian curve, \(\theta\) on the horizontal axis, \(r\) on the vertical axis, then sketch the polar curve by observing the variation in the Cartesian curve.
**Symmetry** can be exploited to reduce the amount of work in graphing:

1. \( F(r, \theta) = F(-r, \theta) \Rightarrow \text{Graph is symmetric about the pole, any part of the graph rotated 180° about the pole is still part of the graph.} \)

2. \( F(r, \theta) = F(r, -\theta) \Rightarrow \text{Graph is symmetric about the polar axis, any part of the graph when reflected about the (horizontal) polar axis is still part of the graph.} \)

3. \( F(r, \theta) = F(r, \pi - \theta) \Rightarrow \text{Graph is symmetric about the ray } \theta = \frac{\pi}{2}, \text{ any part of the graph when reflected about the (vertical) ray } \theta = \frac{\pi}{2} \text{ is still part of the graph.} \)

HW: 11.3 #1, 3, 5, 7, 9, 15, 17, 23, 29, 31, 37, 41