Parametric Curves

Curves described by Parametric Equations

Suppose we have two functions $f$ and $g$ such that $x = f(t)$ and $y = g(t)$. Each value of $t$ determines a point $(x, y)$ in the x-y plane. As $t$ varies, the point $(x, y) = (f(t), g(t))$ traces out a curve $C$ called a parametric curve. The equations $x = f(t)$ and $y = g(t)$ are called parametric equations for the curve $C$ with $t$ as parameter.

If $t$ is time, then $(x, y) = (f(t), g(t))$ can be interpreted as the position of a particle in the x-y plane at time $t$.

For curve $C$ with parametric equations $x = f(t)$ and $y = g(t)$, $a \leq t \leq b$, the point $(f(a), g(a))$ is called the initial point and the point $(f(b), g(b))$ is called the terminal point. We indicate by an arrowhead on the curve the direction in which the curve $C$ is traced as $t$ increases from $a$ to $b$.

A trochoïd is generated by the motion of a point at a distance $t$ from the center of a circle that has radius $r$, $0 < t \leq r$, as the circle rolls along a straight line. A cycloïd is a trochoïd with $t = r$, it is the curve generated by the motion of a point on a circle that rolls along a straight line. A hypocycloïd is generated by the motion of a point on a circle that rolls inside another circle. An epicycloïd is generated by the motion of a point on a circle that rolls outside another circle.

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