Math 462 Homework Assignment #9

Your Name: ________________________________

Due Date: April 10, 2003

1 Let $S$ denote Enneper’s surface

$$\mathbf{x} = \left( u - \frac{u^3}{3} + uv^2, \ v - \frac{v^3}{3} + vu^2, \ u^2 - v^2 \right).$$

Find

(a) the first fundamental form

(b) the area form.

2 Find the first and second fundamental forms of the surface parameterized by

$$\mathbf{x} = u \mathbf{e}_1 + v \mathbf{e}_2 + (u^2 + v^3)\mathbf{e}_3.$$

3 A cone (minus its vertex) is a surface. For the standard cone $z = \sqrt{x^2 + y^2}$, find the Gauss curvature $K$ and mean curvature $H$ using the patch $\mathbf{x} = (u, v, \sqrt{u^2 + v^2}).$

4 Find the second fundamental form for the surface parameterized by

$$\mathbf{x} = \left( v \cos(u), \ v \sin(u), \ v \right).$$

5 Let $c(s)$ be a unit speed curve (i.e., $|c(s)| = 1$) and $\mathbf{x} = c(s) + v\dot{c}(s)$ be the tangent surface of $c$. Show that the first fundamental form is

$$I = (1 + v^2 k(s)^2)ds^2 + 2dsvd + dv^2,$$

where $k(s)$ is the curvature of $c(s)$.

6 Let $S$ be a surface parameterized by

$$\mathbf{x} = (u + v)\mathbf{e}_1 + (u - v)\mathbf{e}_2 + uv\mathbf{e}_3.$$

Find the Gauss curvature and mean curvature at $u = 1, v = 1$. 

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