Math 191 Review for Exam 2  
Fall 2009

1 Find all the points where the tangent line to \( f(x) = x^3 - 6x^2 - 15x \) is horizontal. \( (x = 5, x = -1) \)

2 The height of an object falling due to gravity is given by \( s(t) = s_0 + v_0 t - \frac{1}{2}gt^2 \). Suppose a toy rocket is fired from the ground with an initial velocity of 160 ft/sec.

(a) Determine when the rocket reaches its maximum height and the maximum height. \( (5 \text{ sec}, 400 \text{ ft}) \)
(b) When does the rocket hit the ground? What is its velocity at that time? \( (10 \text{ sec}, -160 \text{ ft/sec}^2) \)

3 Let \( f(x) = e^x \sin x \). Calculate \( f' \) and \( f'' \). \( (e^x(\sin x + \cos x), 2e^x \cos x) \)

4 Use implicit differentiation to calculate \( \frac{dy}{dx} \): \( xy^2 = 1 - \cos(x + y) \). \( \left( \frac{\sin(x+y)-y^2}{2xy-\sin(x+y)} \right) \)

5 For the ellipse: \( 2x^2 + 3y^2 + 2xy = 15 \)

(a) Write an equation for the tangent line to the ellipse at (2,1). \( (y = -x + 3) \)
(b) Find all points on the ellipse at which the tangent line is horizontal. \( \left( \left( \frac{\sqrt{3}}{2}, \sqrt{3} \right), \left( -\frac{\sqrt{3}}{2}, -\sqrt{3} \right) \right) \)

6 Find the derivatives of the following functions.

(a) \( y = \cos^{-1}(t) - \sqrt{1-t^2} \). \( \left( \frac{t-1}{\sqrt{1-t^2}} \right) \)
(b) \( y = e^{\tan^{-1}x^2} \). \( \left( \frac{2xe^{\tan^{-1}x^2}}{1+x^4} \right) \)
(c) \( y = x \sin^{-1}(e^x) \). \( \left( \sin^{-1}(e^x) + \frac{xe^x}{\sqrt{1-(e^x)^2}} \right) \)
(d) \( g(s) = s \cos(e^{3s^2-7}) \). \( \left( \cos(e^{3s^2-7}) - 3s \sin(e^{3s^2-7}) e^{3s^2-7} \right) \)
(e) \( h(x) = \tan^2(xe^x) \). \( \left( 2 \tan(xe^x) \sec^2(xe^x)(e^x + xe^x) \right) \)
(f) \( y = \frac{3 + x^{-3}}{\ln(x+1)} \). \( \left( -\frac{4+3\ln(x+3)x^3}{x^3(\ln(x+1))^2} \right) \)

7 Find an equation of the tangent line at the point \( x = 1 \) to \( y = x(3^x) \). \( (y = 3x - 2) \)

8 Use logarithmic differentiation to find the derivative of \( y = \sqrt{\frac{x(x+5)}{(3x+7)(5x-1)}} \).

\( \left( \frac{dy}{dt} = \frac{1}{2} \sqrt{\frac{x(x+5)}{(3x+7)(5x-1)}} \left( \frac{x}{2} + \frac{1}{x+5} - \frac{3}{3x+7} - \frac{5}{5x-1} \right) \right) \)

9 If a snowball is melting so that its surface area is decreasing at a rate of 0.5 \( \text{cm}^2/\text{min} \), find the rate at which the radius is decreasing when the radius is 4 cm. \( (-\frac{1}{64\pi}) \text{ cm/min} \)

10 As a man walks away from a 12-ft lamppost, the tip of his shadow moves twice as fast as he does. What is the man’s height? \( (6 \text{ ft}) \)

11 Solve problem 17, p. 205.

12 Estimate \( \Delta f \) using Linear Approximation for \( f(x) = \cos x \) at \( a = \pi/4 \) where \( \Delta x = 0.1 \). \( (-\frac{\sqrt{2}}{20}) \)

13 Find the linearization of \( f(x) = \sin^{-1}x \) at \( a = 1/2 \). \( \left( \frac{\pi}{6} + \frac{\sqrt{3}}{8}(x - \frac{1}{2}) \right) \)

14 Find the maximum and minimum values of the following functions on the given interval.

(a) \( f(x) = x^3 - 3x + 1 \), \( [0, 2] \). \( (f(2) = 3, f(1) = -1) \)
(b) \( f(x) = x - \sin x \), \( [0, 2\pi] \). \( (f(2\pi) = 2\pi, f(0) = 0) \)
(c) \( f(x) = xe^{-x} \), \( [0, 2] \). \( (f(1) = 1/e, f(0) = 0) \)
(d) \( f(x) = x^5 - 3x^2 \), \( [-1, 5] \). \( (f(5) = 3050, f(-1) = -4) \)

15 Find the critical points of \( f(t) = 4t - \sqrt{t^2 + 1} \). \( (\text{no critical points}) \)

16 Estimate \( \sin(\pi/4 + \pi/16) \) using the linearization of \( f(x) = \sin x \) at \( a = \pi/4 \). \( \left( \frac{\sqrt{2}}{2}(1 + \frac{\pi}{16}) \right) \)