Problem 1 (5 points) Given that \( \lim_{x \to 4} \frac{x}{x^2 - 1} = \frac{1}{4} \), illustrate the **Precise Definition** of a limit (in other words the \( \varepsilon - \delta \) Definition of a limit) by finding a value of \( \delta \) that corresponds to \( \varepsilon = \frac{1}{8} \).

Problem 2 (15 points) Find the limits, if they exist.

a. \( \lim_{x \to -1} \frac{x^2 - 1}{x + 1} \)

b. \( \lim_{x \to -1} \frac{x^2 + 1}{x - 1} \)

c. \( \lim_{x \to \infty} \frac{x^2 - 1}{3x^2 + 1} \)

Problem 3 (10 points) Compute the derivative of \( f \)

a. \( f(x) = \frac{1}{x^5} - \sqrt[3]{3x^3 - x + 5} \)

b. \( f(x) = \tan^2 x - \tan x^2 \)

Problem 4 (5 points) Find an equation to the tangent line to the graph of \( f(x) = \sqrt{x^2 - 1} \) at the point \( (2, \sqrt{3}) \).

Problem 5 (10 points) (a). Find the intervals on which the function \( f(x) = 2 + 3x - x^3 \) is increasing and decreasing.

(b) Where is \( f \) concave up?

Problem 6 (10 points) Sketch the graph of the function \( f(x) = \frac{2x^2}{9 - x^2} \). This function has the following derivatives: \( f'(x) = \frac{36x}{(9 - x^2)^2} \) and \( f''(x) = \frac{108(x^2 + 3)}{(9 - x^2)^3} \). DO NOT recalculate the derivatives; just USE them!
Problem 7 (10 points) A builder intends to construct a storage shed having a volume of 900 ft$^3$, a flat roof, and a rectangular base whose width is three-fourths the length. The cost per square foot of the material is $4 for the floor, $6 for the sides and $3 for the roof. What dimensions will minimize the cost?

Problem 8 (5 points) Evaluate the integral by interpreting it in terms of an area

$$\int_0^2 \sqrt{(4-x^2)} \, dx$$

Problem 9 (15 points) Evaluate

a. $$\int_1^2 \left( \frac{3}{x^2} - \sqrt{2x + 3} \right) \, dx$$

b. $$\int (\sin x \cos x + \tan x \sec x) \, dx$$

c. $$\int (x \sin x^2 + \sec^2 x) \, dx$$

Problem 10 (5 points) Sketch and find the area of the region bounded by the graphs of $y = x$, and $y = 2 - x^2$.

Problem 11 (10 points) Let $R$ be the region bounded by the graphs of $y = \sqrt{x}$ and $y = x$.

(a) Find the volume of the solid generated by revolving $R$ about the $x$-axis.

(b) Find the volume of the solid generated by revolving $R$ about the $y$-axis.

Bonus Problem (5 points) Calculate the given limit

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i}{n^2}$$