DEPARTMENT FINAL EXAMINATION
MATH 165
ANALYTIC GEOMETRY &
CALCULUS I
FALL 2012

Mark your section here

11650  MWF 9:00 - 10:15 AM  Watt, J.
11651  MWF 10:30 - 11:45 AM  Shen, Z.
11652  MWF 12:00 - 1:15 PM  Harsy-Ramsay
11653  MWF 1:30 - 2:45 PM  Shen, Z.
11654  MW 6:00 - 7:50 PM  Molkov, Y.
11655  TR 6:00 - 7:50 PM  Xu, D.

Directions
1. Place your name and student ID number on this cover sheet, and put a check beside your section number.
2. You will have 2 hours to complete this examination.
3. NO CALCULATORS.
4. Cell phones and PDAs may not be turned on or used during the final.
5. No scrap paper, notes, or books are permitted.

Page  Possible  Score  Sub T
1       10
2       12
3       12
4       12
5       12
6       12
7       16
8       8
9       6
10      8
Total   108

Last Name

First Name

10-Digit University ID
1 (4 points) Prove the statement using the $\epsilon, \delta$ definition of a limit. \( \lim_{x \to 2} (3x - 4) = 2 \).

2 (6 points) Let \( f(x) = \sqrt{x - 12} \). Find the absolute maximum value and the absolute minimum value of \( f \) on \([0, 16]\).
3 (12 points) Find the limit if it exists. Explain why if it does not exist.

(a) \( \lim_{x \to 1} \frac{x + 1}{x - 1} \)

(b) \( \lim_{x \to 1} \frac{x^2 + 3x + 2}{x^2 - 2x - 3} \)

(c) \( \lim_{x \to 1} \frac{\sqrt{3x^2 + 6}}{x^2 + 1} \)

(d) \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \), where \( f(x) = \sqrt{x + 1} \).
4 (12 points) Compute the derivatives of the following functions.

(a) \( f(t) = t^{1/3}(t^2 + t + t^{-1}) \)

(b) \( g(x) = (x^2 + 1)^3(x^2 + 2)^5 \)

(c) \( f(x) = \int_{1}^{\sqrt{x}} t \sin(t^2) \, dt \)
5 (6 points) Use implicit differentiation to find an equation of the tangent line to the curve $7x^3 + xy^{20} = (x^2 + y^2)^3$ at $(1, 1)$.

6 (6 points) Sketch the region enclosed by the curves: $x - y^2 + 2 = 0, x = y$. Find the area of the region.
7 (12 points) Let \( f(x) = x^3 - 2x + 1 \).
(a) find critical numbers,
(b) determine intervals on which \( f \) is increasing or decreasing,
(c) find local maximum values and local minimum values,
(d) determine the intervals on which the graph is concave up or concave down,
(e) Sketch the graph.
8 (6 points) Find the point on the line \( y = x + 3 \) that is closest to the point \((-1,0)\).

9 (6 points) Evaluate \( \int_0^2 (3x^2 - 2x)\,dx \) using the Riemann sum.
10 (16 points) Evaluate the following integrals:

(a) $\int (2x - 3)(1 - 4x)dx$.

(b) $\int_{0}^{1} \frac{x}{\sqrt{1-x^2}}dx$.

(c) $\int \frac{x^2 - 3\sqrt{x}}{x^4}dx$.

(d) $\int x \sin(x^2)dx$. 
11 (8 points) Let $A$ be the region bounded by the graphs of $y = x^2$ and $y = 2x$. Set up (but do not compute) the integral to find the volume of the solid generated by revolving $A$ about

(a) the $y$-axis.
(b) the line $y = -1$.

Indicate the methods you use.
12 (6 points) A cable that weighs 10 lb/ft is used to lift 100 lb of coal up a mine shaft 200 ft deep. Find the work done.
Bonus

13 (4 points) A street light is mounted at the top of a 20-ft-tall pole. A man 6 ft tall walks towards the pole with a speed of 2 ft/s along a straight path. How fast is the length of his shadow decreasing when he is 20 ft from the pole?

14 (4 points) Evaluate \( \int \frac{\sqrt{x}}{(\sqrt{x} - 4)^2} \, dx \).