Math 164 Final Exam, Fall 2008

1. Find $y'$ if

(a) $y = 2^x \ln 4x$

(b) $y = \arctan e^{x^2}$
2. Find

$$\lim_{x \to \infty} \left( 1 - \frac{1}{x} + \frac{1}{x^2} \right)^x$$
3. Evaluate the following integrals.

(a) \[ \int x \ln x \, dx \]

(b) \[ \int \frac{x^2}{\sqrt{(x^2 + 4)^3}} \, dx \]
4. Evaluate the integral.

\[
\int_0^1 \frac{dx}{\sqrt{1-x}}
\]
5. Find an equation of the line which is tangent to the parametric curve \( x = 2 \cos t, y = \sin t \), with \( 0 \leq t \leq \frac{\pi}{2} \), at the point \( (\sqrt{2}, \frac{\sqrt{2}}{2}) \).

6. Find the length of the arc of the curve \( g^2 = 2x^3 \) from point \( (0, 0) \) to point \( (2, 4) \).
7. Set up, **but do not evaluate**, an integral which represents the area of the region which lies between the curves:

\[ r = 1 + \sin \theta \]
\[ r = 1 - \sin \theta \]
8. Determine the limit of the following sequence, if it exists. Make sure and show all work involved in the calculation.

\[
\lim_{n \to \infty} \left\{ \frac{1 - 3n}{4n + 1} \right\}_{n=1}^{\infty}
\]

9. Determine whether or not the following series is conditionally convergent, absolutely convergent, or divergent. State carefully which test you are using.

\[
\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^2 + 1}
\]
10. Determine whether or not the following series converges or diverges. State carefully which test you are using.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n + 1}}$$

11. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^2}{10^n(x + 7)^{n+1}}$$
12. Find the Maclaurin series for

\[ \frac{2x}{1 + x^2} \]

**Bonus** Determine whether or not the following series converges or diverges. If it converges, find the sum of the series.

\[ \sum_{n=0}^{\infty} \frac{2^n}{n!} \]