NAME:

MATH 261  
TEST 2, 30 points

SHOW YOUR WORK PLEASE

Problem 1 3 points Use implicit differentiation to find \( \partial z/\partial x \) and \( \partial z/\partial y \)

\[
\sin(xy^z) = x + 2y + 3z
\]

ANSWER

Problem 2 3 points. Find equations of the tangent plane to the given surface at the specified point.

\[
yz = \ln(x + z), \quad (0, 0, 1)
\]

ANSWER
**Problem 3** 3 points. Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $P(1,-1,3)$ in the direction of $Q(2,4,5)$.

**ANSWER:**

**Problem 4** Find the absolute maximum and absolute minimum values of $f$ on the set $D$.

$$f(x, y) = x^2 + 2y^2 - x \quad D = \{(x, y) | x \geq 0, y \geq 0, \ x^2 + y^2 \leq 4\}$$

**ANSWER**
Problem 5 3 points. Find the local maximum and local minimum and saddle point(s) of the function.

\[ f(x, y) = x^4 + y^4 - 4xy + 2 \]
Problem 6 3 points. Use Lagrange multipliers to find the max and min values of $f$ subject to the given constrain(s)

$$f(x, y, z) = x^2 y^2 z^2; \quad x^2 + y^2 + z^2 = 1$$

ANSWER:
Problem 7 3 points. Evaluate the double integral $\int_D 2xy \, dA$, D is the triangular region with vertices (0,0), (1,2) and (0,3).

Answer

Problem 8 3 points. Sketch the region of integration and change the order of integration.

$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y) \, dy \, dx$$

Answer:
Problem 9 3 points. Use polar coordinates to find the area of the region OUTSIDE the cardioid $r = 1 + \cos \theta$, and INSIDE the circle $r = 3 \cos \theta$. 

Answer
Problem 10 3 points. Evaluate the integral by reversing the order of integration

\[ \int_0^8 \int_{\sqrt[4]{y}}^2 e^{x^4} \, dx \, dy \]

Answer

BONUS Problem 3 points. Suppose \( f \) is a differentiable function of one variable. Show that all tangent planes to the surface \( z = x f(y/x) \) intersect in a common point.