I would like to propose one more approach to well-known general problem of S.Kakutani and J.Doob of unification both Martingale convergence and Ergodic theorems.

Suppose that $(\Omega, F, \lambda)$ is a probability space, $T$ is its automorphism, $f \in L_1(\Omega)$. Let $\{F_n\}_{1 \leq n \leq \infty}$ be a monotone sequence of $\sigma$-subalgebras $F$ such that $F_n \uparrow F_\infty$ (or $F_n \downarrow F_\infty$) as $n \to \infty$. Set

$$A_nf = \frac{1}{n} \sum_{k=0}^{n-1} f \circ T^k, \quad f^* = \lim_{n \to \infty} A_nf, \quad f^*_\infty = E(f^*|F_\infty).$$

**Problem.** To prove convergence a.e. of conditional expectations $E(A_nf|F_n) \to f^*_\infty$ as $n \to \infty$.

**Comments.** In the degenerate case $F_n \equiv F$ it will be $E(A_nf|F_n) = A_nf$, and we have usual Ergodic theorem. In the case $T \equiv \text{id}$ it will be $E(A_nf|F_n) = E(f|F_n)$, and we have Martingale convergence theorem for reversed martingale (if $F_n \downarrow$) or for regular straightforward martingale (if $F_n \uparrow$).

**Known.** It is proved already (Math. Notes, 1998, 64:2, P.266–269) that proposed statement holds for all $f \in L\log L$. Also, it is proved convergence in $L_p$ for every $p \in [1, \infty)$ (Ibid).

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