MATH 118           Extra Problems For Chapter 4

Note: Make sure you do the following problems and turn them in with the homework

Sections 4.1, 4.2

1. The letters A, B, C, D, E, F and G are to be randomly arranged in a row. Find the probability that:
   a) the arrangement is ABCDEFG
   b) the first 3 letters, in order, are ABC.
   c) the letters ABC appear together in that order.
   d) the letters A, B and C appear together in any order.

2. A student has errands at the bank, grocery store, and the music store. The student makes random choice of
   the order in which to do the errands. Find the probability the bank is visited before the grocery store.

3. Five students, Ann, Bill, Chuck, Debra and Ed are randomly assigned seats from the seats numbered 1
   through 10. What is the probability that Ann is assigned either #1 or #10?

4. Suppose your checkbook balance is at least $10.00 and no more than $99.99. Assuming that all balances
   are equally likely to occur, what is the probability that your balance is an exact number of dollars, for
   example, $12.00?

5. A bucket contains 3 red balls, 4 white balls, and 5 blue balls. Two balls are selected at random and the
   colors are noted. What is the probability that at least one ball is not white and at least one is not blue?

6. A pair of dice are rolled and the number of dots on the top are noted. Assume both dice are fair, what is
   the probability that:
      a) the numbers are the same on the two dice?
      b) the sum of the numbers is even?

7. A bucket contains 10 audio tapes in which 3 are defective. Three tapes are selected at random, what is the
   probability that:
      a) at most one is defective?
      b) at least one is defective?

8. A poker hand consists of 5 cards selected at random from an ordinary deck. Find the probability that the
   poker hand contains:
      a) How many poker hands contain 5 consecutive cards of the same suit (Assume that an Ace can be
         used either high or low; that is both A-K-Q-J-10 or 5-4-3-2-A)
      b) A pair is a 5-card hand in which just 2 of the cards are the same denomination and the others are
         not, such as Q-Q-5-4-2. How many are there?

Answers (not in order):
    [a) 98/120  b)17/24] ; [ 2. P(9,4)/P(10,5) = 1/5 ] ; [25/33] ; [a) 1/6  b) 18/36] ; [1/2] ; [90/9000 = 1/100] ;
    [ a) (4)(10)/C(52,5) b) 1098240] ; [a) 1/5040  b) 24/5040  c) 120/5040  d) 720/5040]

9. Let A, B, C and D be events which form partition of a sample space S. If P[A]=P[B], P[C]=2P[A], and
   P[D]= 2P[C]. Find: P[A], P[A∪B]

10. Suppose E, F and G are events in a sample space S with P[E] = 0.7, P[F] = 0.4, and P[G] = 0.2, P[E∪F] = 1, and F∩G = ∅ . Find P[E∩G].

11. Suppose A and B are events in a sample space S with P[A] = 0.5, P[B] = 0.3. If B ⊂ A, find P[A∪B]

12. Suppose A and B are events in a sample space S with P[A|B] = 0.75, P[A] = 0.5, and P[B'] = 0.75.
    Find P[B|A].

13. Let A and B be events in a sample space S with P[A] = 0.6, P[A ∩ B'] = 0.2, and P[B] = 0.5. Find P[A|B].
14. The events $E$ and $F$ are independent in a sample space $S$, $P(E)=0.3$, and $P(F)=0.6$. Find $P(E|F')$.

15. Let $A$ and $B$ be events such that $P(A \cup B) = 0.8$ and $P(A)=0.6$. What is the $P(B)$ if $A$ and $B$ are independent?

16. Let $A$ and $B$ be events such that $P[A] = 0.6$, $P[A \cup B] = 0.9$, and $P[B|A]=0.4$. Find $P[B]$.

**Answers #9 - 16 (not in order):** [0.8] ; [0.375] ; [0.5] ; [0.2] ; [1/8, 1/4] ; [0.3] ; [0.54] ; [0.5]

17. A student attends mathematics class with probability 0.6, skips accounting class with probability 0.3, and attends both with probability 0.5. Find the probability that she attends at least one class.

**Answer:** 0.8

18. Half of student attending the business class read the *Wall Street Journal*, and 2/3 of the students read *The Indianapolis Star*. Reading those newspapers are independent, what is the probability that a randomly selected student read exactly one of these newspapers?

**Answer:** 1/2

19. A television set which sometime malfunctions has the following probabilities:
   - A clear picture = 0.7
   - Good color = 0.6
   - Good sound = 0.5
   - Both clear picture and good sound = 0.4
   - Both clear picture and good color = 0.5
   - Both good sound and good color = 0.4
   - All = 0.3

Find the probability of exactly one of the three characteristics.

**Answer:** 0.1

20. There are 2 white balls, 2 red balls, and 3 blue balls in a box. Two balls are selected and their colors are noted. Find the probability that neither is white given that neither is blue.

**Answer:** 1/6

21. A box contains 1 green, 2 blue, and 2 red balls. Two balls are selected and their colors are noted. Find the probability that one is red and one is blue given that they are not same color.

**Answer:** 1/2

22. In a lab, there are 12 mice: 2 gray females, 3 gray males, 3 white females, and 4 white males. Two mice are selected and their color and sex are noted, find the probability that:
   a) both are females given that both are gray
   b) 1 mouse is a female and 1 mouse is male given that both are gray
   c) at least 1 mouse is male given that both are white
   d) they are opposite sex given that they are opposite color

**Answer:** a) 1/10 b) 6/10 c) 6/7 d) 17/35

23. In a lab, there are 8 mice: 2 gray males, 3 gray females, and 3 white males. Two mice are selected and their color and sex are noted. Find the probability that at least one is male, given that exactly one is gray.

**Answer:** 1

24. A pair of dice are rolled and the number of dots on the top are noted. Assume both dice are fair, what is the probability that:
   a) the sum is 6 given that at least one of the numbers is less than 3?
   b) the sum is 9 given that at least one of the numbers is odd?
   c) the sum is 8 given that at least one of the numbers is even?
   d) at least one of the numbers is even given that the sum is 8?
   e) exactly one die shows 4 given that the sum of the numbers is 7?
   f) the sum of the numbers is 7 given that at least one die shows 4?

**Answer:** a) 4//20 b) 4/27 c) 3/27 d) 3/5 e) 1/3 f) 2/11
Sections 4.3, 4.4

25. A firm has new test to detect lung cancer. It researches the values of the test on thousands of people and find that if a person has lung cancer, then the test will detect it in 97% of the cases; and if a person does not have lung cancer, it will still show a positive result in 2% of the cases. Later medical records show that 9% of the group actually did have lung cancer.
What is the probability that a person who tests:
   a) positive actually has lung cancer?
   b) positive actually does not have lung cancer?
   c) negative actually has lung cancer?

\[ \text{Answer: a) } 0.8275 = 82.75\% \]
\[ b) 17.25\% \]
\[ c) 0.003 = 0.3\% \]

26. Consider the following three-stage experience which uses boxes, urns, and colored balls. An experiment consists of selecting at random one of the two boxes X or Y, then select an urn, then select a ball from that urn and note the color of the ball. If the ball selected is red, what is the probability that box X was selected? \((\text{Draw the tree to solve the problem})\)
\((r=\text{red}, w=\text{white}, b=\text{blue})\)

\[ \text{Answer: } \frac{70}{121} \]

27. Consider 2 bags, each of which contains 3 colored balls. Bag \(a\) contains 2 red and 1 white ball, and bag \(b\) contains 1 red, 1 white, and 1 blue ball. An experiment consists of randomly drawing ball from bag \(a\), noting its color, placing the ball in bag \(b\), and then randomly drawing a ball from bag \(b\) and noting its color. Draw the tree diagram and find the probability of obtaining 2 red balls.

\[ \text{Answer: } \frac{1}{3} \]

28. A new small business makes a profit the first year with probability 0.2. After the first year it makes a profit with probability 0.6 if it made a profit in the preceding year, and it makes a profit with probability 0.4 if it did not make a profit in the preceding year. Find the probability that it makes a profit for exactly 2 of the first 3 years.

\[ \text{Answer: } 0.272 \]

29. A box contains 1 red, 2 green, and 3 yellow balls. A ball is selected at random and its color is noted. If it is green it is replaced; otherwise it is not. A second ball is selected and its color is noted. If the second is green, find the probability that the first was green also.

\[ \text{Answer: } \frac{5}{17} \]

30. A selection is made between two coins and the selected coin is flipped twice. One of the coins is fair and the other is unfair with \(P[H]=\frac{2}{3}\). If the result of the experiment is one head and one tail, what is the probability that the unfair coin was selected?

\[ \text{Answer: } \frac{8}{17} \]
31. One third of students in a class are women. Women are twice as likely to bring their lunch to school than men. Half of the women bring their lunch. If a student selected brought lunch to school, find the probability that this student is a man

Answer: 1/2

32. In a lab, there are 8 mice: 1 gray male, 4 white females, and 3 white males. Two mice are selected one after the other without replacement and their color and sex are noted, find the probability that:
   a) the first is was a female given the second was a male
   b) the first is was a gray given the second was a male

Answer: a) 4/7  b) 3/28

33. A multiple-choice test contains 12 questions, with 4 choices for each answer. If a student guesses the answers, find the probability that he will get at least 1 correct answer.

Answer: 1-[C(12,0)(0.25)^0(0.75)^12] = 0.9683

34. For a Bernoulli process with n=5 and p = 0.6, find the probability of at least 4 successes or at least 3 failures.

Answer: 1-[C(5,3)(0.6)^3(0.4)^2] = 0.6544

35. A coin is weighted so that a head is twice as likely to occur as is a tail. The coin is flipped four times, find the probability that both heads and tails occur

Answer: 1 - (2/3)^4 - (1/3)^4

36. A coin is weighted so that a head is twice as likely to occur as is a tail. The coin is flipped three times, find the probability that there is exactly one head given that both heads and tails occur

Answer: 1/3

37. Each morning Tom decides whether to attend economic class. He attend with probability 0.6, and each decision is independent of what he has done in the past (Bernoulli process). Find the probability that he attends at least 6 of 10 classes given that he attends at least one but not all of the 10 classes.

Answer: 0.6308

38. In a garage, a car out of 10 is defective and will not start. At different times, 3 individuals randomly select cars for test drives and then return the car to the garage. What is the probability that at least one of them selects a defective car?

Answer: 0.271

39. In a city 50 % of the voters are Democrats, 25 % are independent, and 25% are Republican. If 10 people are asked at random, what is the probability that at least 7 are Democrats?

Answer: 0.171

40. A die is rolled 10 times. A roll is called a success if either a 1 or a 2 comes up. What is the probability that there exactly 3 successes or exactly 3 failures in 10 rolls?

Answer: C(10,3)(1/3)^3(2/3)^7 + C(10,3)(1/3)^7(2/3)^3

41. A multiple-choice test contains 10 questions, with 5 choices for each answer. Two students guess the answers, find the probability that:
   a) each will get exactly 4 correct answers.
   a) both answered the same 4 questions correctly

Answer: a) [210(1/5)^4(4/5)^6]^2  b) 210[(1/5)^4(4/5)^6]^2

42. A coin is with P[H] = x > 0 is flipped 10 times. If the probability of exactly 4 heads is equal to the probability of exactly five heads, what is x ?

Answer: 5/11