SECTION 4.1
The probability or (likelihood) of the occurrence of an event is found in two methods:

(I) Relative Frequency Method:

Ex: In a survey of 100 people, it was found that 57 watch the late news.
P(E): probability that a person watches late news P(E) = 57%
P(E'): probability that a person does not watch late news P(E') = 43%

\[ P(E) = \frac{m}{n} \]

m: the number of times that the event actually is observed.
n: the number of times the experience is attempted

\[ P(E) + P(E') = 1 \]

(II) Deductive Method: (the method of this course)

Outcomes: a particular result of an experience.
Sample Space: the set of all possible outcomes of an experiment.

Ex: * By rolling a die once, the sample space is \( S = \{1, 2, 3, 4, 5, 6\} \)
  * By flipping a coin twice, the sample space is \( S = \{HH, HT, TH, TT\} \)

Equally Likely: When each of the outcomes of an experiment has the same probability of occurring (fair die, fair coin,....)

Probability of an event \( P(E) = \frac{n(E)}{n(S)} \)

\( n(E) \): number of outcomes where the event occurs.
\( n(S) \): total number of possible outcomes in the sample space.

Ex1: By rolling a pair of dice, find all outcomes of sums and the probability of each:

<table>
<thead>
<tr>
<th>Sum of</th>
<th>By</th>
<th># of ways</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11</td>
<td>1</td>
<td>1/36</td>
</tr>
<tr>
<td>3</td>
<td>21, 12</td>
<td>2</td>
<td>2/36</td>
</tr>
<tr>
<td>4</td>
<td>22, 31, 13</td>
<td>3</td>
<td>3/36</td>
</tr>
<tr>
<td>5</td>
<td>32, 23, 41, 14</td>
<td>4</td>
<td>4/36</td>
</tr>
<tr>
<td>6</td>
<td>33, 42, 24, 51, 15</td>
<td>5</td>
<td>5/36</td>
</tr>
<tr>
<td>7</td>
<td>43, 34, 61, 16, 52, 25</td>
<td>6</td>
<td>6/36</td>
</tr>
<tr>
<td>8</td>
<td>44, 53, 35, 62, 26</td>
<td>5</td>
<td>5/36</td>
</tr>
<tr>
<td>9</td>
<td>54, 45, 63, 36</td>
<td>4</td>
<td>4/36</td>
</tr>
<tr>
<td>10</td>
<td>55, 64, 46</td>
<td>3</td>
<td>3/36</td>
</tr>
<tr>
<td>11</td>
<td>56, 65</td>
<td>2</td>
<td>2/36</td>
</tr>
<tr>
<td>12</td>
<td>66</td>
<td>1</td>
<td>1/36</td>
</tr>
</tbody>
</table>

Sum = 36 \[ \frac{36}{36} = 1 \]

Total number of events is 36 or 6x6 = 36

The sum of probabilities for all outcomes is always = 1
**Ex2:** By rolling a pair of dice, find the probability of:
   a) getting the sum of 6.  
   b) not getting the sum of 6

**Ex3:** By selecting 3 cards, find the probability of getting:
   a) same colors.  
   b) a pair of kings (such as K,K,2 )  
   c) a pair  
   d) one ace and the others are the same suit as the ace.

**Ex4:** By selecting 5 cards, find the probability of getting:
   b) exactly 3 Aces  
   c) all face cards  
   c) same suit

**Ex5:** In a box there are 15 Science books and 10 History. If 7 books are selected at random (equally likely), find the probability of getting at least 1 Science book.

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

- \(P(A \cup B)\) = probability of \(A\) or \(B\) (either \(A\) or \(B\) or both)
- \(P(A \cap B)\) = probability of both \(A\) and \(B\)
  - \(= P(A).P(B)\); if they are independent (will be in section 4.2)
  - \(= 0\); if they are mutually exclusive (disjoint)
- \(P(A \cup B)' = probability\ of\ neither\ \(A\ nor\ \(B\ &= 1 - P(A \cup B)\)

**Ex6:** Out of 90 students surveyed, 30 took Math, 40 took English and 10 took both. What is the probability that a student took:
   a) English and Math  
   b) neither English nor Math.

**Ex7:** The probability that Bob will pass the Math course is 0.6, and that he will pass the English course is 0.7. If the probability that he will pass both of them is 0.4, find the probability that:
   a) he will pass at least one course.  
   b) he will not pass any of the courses  
   c) he will pass either course but not both (only one)

**Ex8:** A survey in a college found that 40% passed the Math test , 70% passed the English test and 10% passed neither test. What is the probability:
   a) of students that passed both test?  
   b) of students that passed one subject only?

**The Odds:**

- **Given the probability, find the odds:** If the probability of an event \(E\) is \(P\), then

\[
\text{Odds for the event } = \frac{p}{1-p} \quad \text{Odds against the event } = \frac{1-p}{p}
\]

**Ex9:** The probability for winning a game is \(P(E) = 7/12\). What is the odds:
   a) for winning \((\text{ans: } 7/5)\)  
   b) against winning \((\text{for loosing})\) \((\text{ans: } 5/7)\)

- **Given the odds, find the probability:** If the odds for making an event \(E\) are \(a\) to \(b\), then:

\[
\text{Probability of } (E) = \frac{a}{a+b} \quad \text{Probability of } (E') = \frac{b}{a+b}
\]

**Ex10:** The odds for winning a game is 7/5. What is the probability of:
   a) winning \((\text{answer: } P(E) = 7/12)\)  
   b) loosing \((\text{answer: } P(E') = 5/12)\)
SECTION 4.2: Conditional Probability & Independence

\[
P(E|F) = \frac{P(E \cap F)}{P(F)} \quad \text{and} \quad P(F|E) = \frac{P(E \cap F)}{P(E)}
\]

\(P(E|F)\): probability of \(E\) given \(F\), or probability of \(E\) knowing \(F\).

\(P(F|E)\): probability of \(F\) given \(E\), or probability of \(F\) knowing \(E\).

The two events \(E\) & \(F\) are **independent** if: \(P(E \cap F) = P(E). P(F)\)

**Ex11:** If \(P(E) = 2/3\), \(P(F) = 5/8\) and \(P(E \cap F) = 5/12\), are \(E\) & \(F\) independent?

**Ex12:** If \(P(E) = 0.5\), \(P(F) = 0.02\) and \(P(E \cap F) = 0.2\), are \(E\) & \(F\) independent?

**Ex13:** In a survey of 100 people, it was found that:

<table>
<thead>
<tr>
<th>Married  ((R))</th>
<th>Divorced  ((D))</th>
<th>Singles ((S))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male ((M))</td>
<td>25</td>
<td>7</td>
</tr>
<tr>
<td>Female ((F))</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

If one person is selected, find the probability that this person is:

- a) male, female, married, divorced
- b) male and married, male and divorced, female and married
- c) male, given he is married
- d) divorced given the person is female
- e) divorced given the person is male

**Ex14:** A pair of dice are rolled and the numbers are noted. What is the probability that:

- a) the sum is 8 given that the second die shows 3
- b) the sum is 6 given that both are odds.

**Ex15:** A box with 7 red balls, 5 white balls and 4 blue balls. 3 are selected at random, find the probability that:

- a) they are red given that they are of the same color.
- b) one is white given that at least one is white.

**Ex16:** There are 7 women and 5 men in a room in which 3 will be selected at random. Find the probability that:

- a) all are women given that they are of the same gender.
- b) at least 1 is a man and at least 1 is a woman given that the team contain at least 1 man.

**Ex17:** A committee consists of 6 Democrats and 5 Republicans. Three of the Democrats are men and three of the Republicans are men. If 2 people are selected, find the probability that they are:

- a) Republican, given they are men.
- b) opposite gender, given they are Republican.

**Ex18:** The probability that Mike will go to college is 0.4 and that he will join the army is 0.5. Find the probability that he will go to either one if:

- a) the two events are independent
- b) the two events are mutually exclusive.

**Ex19:** Mike and Bob do not know each other. The probability that Bob will pass the math course is 0.3 and that Mike will pass it is 0.6. What is the probability that neither one passes the Math class? (Solve this problem in two different ways, then solve it again graphically)
**SECTION 4.3 Bayes Theorem**

Use the tree method when you have an experiment which consists of a sequence of sub-experiments.

**Ex20:** Two people will be selected without replacement out of 7 women and 2 men.
   a) draw the tree and show all the probabilities.
   b) find the probability that 2 women are selected
   c) find the probability that 2 of the same gender are selected

   **NOTE:** By using the tree method:
   a) the sum of all path probabilities must be = 1
   b) the sum of probabilities of all branches from one node = 1
   c) the sum of all path probabilities that branches from a given node must be equal to the probability reaching that node.

**Ex21:** Solve Ex20 again but by using the probability formula (Section 4.1):

   **Answer:**
   
   b) \( P = \frac{C(7,2)}{C(9,2)} = \frac{21}{36} \)
   c) \( P = \frac{C(7,2) + C(2,2)}{C(9,2)} = \frac{22}{36} \)

**Ex22:** Repeat Ex20 but this time 3 people are selected without replacement, and find the probability of:

   a) exactly 2 are women
   b) at least 2 are women
   c) 1 man given that at least 2 are women
   d) two women given that the first is a woman

**Ex23:** At a state university, 60% are undergraduates, 35% graduates and 5% are in special program. Also, 20% of the undergraduates are married, 40% of the graduates are married and 70% of the special program are married. Draw the tree and find the following probabilities that:

   a) a selected student is married and undergraduate
   b) a selected student is married
   c) a selected married student is an undergraduate

**Ex24:** A fair coin is flipped until 2 heads or 3 tails appear. Draw a tree and determine all probabilities.

**Ex25:** A box contains 10 good parts and 3 defective parts, if parts are selected without replacement one after another until either 2 defective parts are found or four are selected. Draw the tree and show the probabilities, and find the probability that at least 2 are good.

**Ex26:** In a certain class, there were 10% freshman, 30% sophomores, 40% juniors and 20% seniors. Past experiences show that 20% of freshmen, 40% of sophomores, 30% of juniors and 10% of seniors get A. If one student was selected at random:

   a) find the probability that this student got an A
   b) if the student found to be an A student, find the probability that this student was a junior.

**Ex27:** Two groups of students applied for a job, graduate group (4 women and 6 men) and undergraduate group (3 women and 5 men). The company would flip an unfair coin in which \( P(H) = \frac{2}{3} \), if it is a head then the graduate group will be selected and a student from that group will be selected.

   a) find the probability that a man is selected
   b) if the person selected was a man, find the probability that he is from the undergraduate group

**Ex28:** Box A contains 4 white books and 6 red books.
   Box B contains 3 white books and 2 red books.
   Box C contains 2 white books and 4 red books.

   a) if a box was selected and then a book was selected, what is the probability that this book is white
   b) if the book selected was white, what is the probability that this book was from box B?
**Ex29:** A hotel company had an analysis of people who cash personal checks at the hotel. People who never wrote a bad check are considered good credit. The following facts are known:
* of all check-cashers, 90% have good credits
* 2/3 of the good credits customers have Indiana driver's license
* 1/4 of the bad credit customers have Indiana driver's license

If one person was selected at random with Indiana driver's license, find the probability that:
- a) this person is a good credit customer?
- b) this person is a bad credit customer?

**Ex30:** An airline company is planning to screen all employees for the use of illegal drugs. The test has two results positive or negative. Positive result indicates that illegal drugs were used while negative result indicates no illegal drugs were used. The lab that is doing the test found from previous record that:
- If a person did use illegal drugs, the test will detect (show positive result) in 96% of the cases (this means a false negative of 4% because it shows negative in 4% even though the person is a drug user)
- If a person did not use illegal drugs, the test will still show positive result in 10% of the cases (this means a false positive of 10%)

At the end of the test, it was found that 5% of the employee did actually used illegal drugs. What is the probability that a person who tests:
- a) positive actually did use illegal drugs?
- b) positive actually did not use illegal drugs?
- c) negative actually did use illegal drugs?
- d) negative actually did not use illegal drugs?

**Ans:**
- a) \( \frac{(5)(96)}{(5)(96) + (95)(10)} = 0.336 = 33.6\% \)
- b) 1 - 0.336 = 0.66
- c) \( \frac{(4)(5)}{(4)(5) + (9)(95)} = 0.0023 = 0.23\% \)
- d) 1 - 0.0023 = 0.998 = 99.8\%

**SECTION 4.4 (Bernoulli Trials)**

**Ex31:** The probability that a team will win a game is 60%. Find the probability that the team wins:
- a) a game out of 2
- b) the first 2 games out of 3
- c) 2 games out of 3
- d) 2 games out of 4

**Bernoulli trial:** (repeated events) is applied when:
1) each event has two outcomes only, (win, loose); (pass, fail)...
2) the sum of the two probabilities for the two outcomes is 1
3) the events are independent
4) the probability in the repeated events is the same

\[
P = C(n,r) \cdot p^r \cdot q^{n-r} \quad (q = 1 - p)
\]

\( p: \) probability of success (what we are looking for)  \( n: \) total number of trials
\( r: \) number of successes (number of events of what we are looking for)

**Ex32:** the probability of winning a game is 60%, find the probability that the team wins:
- a) 5 games out of 8
- b) at least 6 games out of 8
- c) a least 2 games out of 8

**Ex33:** Two candidates, a Republican and a Democrat, are running for governor. Opinion has it that the Republican is a 60% favorite to win the election. Twelve people are interviewed on the sidewalk, find the probability that:
- a) at least 3 of them prefer the Republican
- b) no more than 10 of them prefer the Democrat.

**Ex34:** By taking a test of 20 questions, each question has 4 choices for an answer and only one answer is correct. If a student is answering the questions by guessing, find the probability that he gets at least 3 correct questions