A set is a collection of items, referred to as the elements of the set.

Example 1: \[ A = \text{Northwest States} = \{WI, MN, ND, MT, ID, WA\} \]

The set represents a group of states in which each state is an element that is included in the set.

\[ ID \in A \; ; \quad \text{also} \quad MN, ND \in A \]

But \[ IN \notin A \quad (\text{Indiana is not an element of set } A) \]
Example 2: If \( A = \{a, c, d, e, f\} \) and 
\( B = \{b, c, d\}; \quad C = \{a, b, d\}; \quad D = \{a, b, d, g\} \)

\( B \subseteq A \); \( B \) contained by \( A \), or \( B \) is subset of \( A \).

(each element of \( B \) is included in \( A \))

\( C \subseteq A \); but \( D \not\subseteq A \) because \( g \) is not included in \( A \)

Important note: which of the following is correct and why?:

a) \( b, c \in A \)
b) \( b, c \subseteq A \)
c) \( \{b, c\} \subseteq A \)
d) \( \{b, c\} \in A \)
Set-Builder Notation:

Example 3: \( I = \{ x \mid x \text{ is an integer between } 2 \text{ and } 8 \} = \{2, 3, 4, 5, 6, 7, 8\} \).
The vertical line \( | \) is read “such as”

Example 4: \( I = \{ x \mid x \text{ is even and } 1 < x < 10 \} = \{2, 4, 6, 8\} \)

# of Subsets:

Example 5: If \( A = \{ A, B \} \); \((Art and Biology)\)
How many decisions can be made regarding taking any of the above courses?

Example 6: If \( A = \{ A, B, C \} \); \((Art, Biology and Computer)\)
How many decisions can be made regarding taking any of the above courses?
<table>
<thead>
<tr>
<th># of elements</th>
<th># of subsets</th>
<th>Example</th>
<th>Subsets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>$A = {a}$</td>
<td>${a}, {\emptyset}$</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>$A = {a, b}$</td>
<td>${a}, {b}, {a, b}, {\emptyset}$</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>$A = {a, b, c}$</td>
<td>${a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {\emptyset}$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Cardinality**: the number of elements in a set.

Example 7: If \( A = \{a, b, c\} \); \( n(A) = 3 \)

**Universal Set** \( U \): The overall set where all other sets are subsets of it.

Example 8: \( U = \{\text{IUPUI students}\} \) with the following subsets:
- \( B = \{\text{Business students}\} \)
- \( F = \{\text{Fresmen students}\} \)
- \( R = \{\text{Resident students}\} \)
- \( S = \{\text{Senior students}\} \)

All of the above are subsets of the universal set \( U \).

**Complement of a set**: (what is missing from a subset compared to the universal set)

Example 9: \( U = \{a, b, c, d, e, f, g, h\} \); \( A = \{a, c, f\} \), \( B = \{b, c, g, h\} \)

Both sets \( A \) and \( B \) are subsets of the universal set \( U \) where:

- \( A' = \{b, d, e, g, h\} \), the elements missing from \( A \)
- \( B' = \{a, d, e, f\} \), the elements missing from \( B \)
Section 2.2: Set Operations

Example 1: Let \( U = \{ a, b, c, d, e, f, g, h, i \} \) with the following subsets

\[
A = \{a, b, d, e\}, \quad B = \{b, c, e, f, g\}, \quad C = \{e, f, h, i\}
\]

Find the following:

a) \( A' \)

b) \( B' \)

c) \( A \cup B \): The union of \( A \) and \( B \) is the set of all elements that are in \( A \) or \( B \) (or both)

d) \( A \cap B \): The intersection of \( A \) and \( B \) is the set of all elements that are in \( A \) and \( B \).

e) \( A \cap (B \cup C) \)

f) \( (A \cap B) \cup C \)
Example 1 Cont.: Let $U = \{a, b, c, d, e, f, g, h, i\}$ with the following subsets

$$A = \{a, b, d, e\}, \quad B = \{b, c, e, f, g\}, \quad C = \{e, f, h, i\}$$

g) $(A - B)$: What is in $A$ and not in $B$

h) $(B - A)$: What is in $B$ and not in $A$

i) $(U - A)$: What is $U$ and not in $A$, which is the same as $A'$

Example 2: If $A = \{1, 2, 3\}$, $B = \{5, 6, 7\}$, $C = \{2, 4\}$

Find the following

a) $A \cup B$

b) $A \cap B$

c) $A - B$

d) $A \times C$ (Cartesian product)

e) $C \times A$
Section 2.3: Venn Diagram

Example 3: If \( U = \{a, b, c, d, e, f, g, h, i\} \) and \( A = \{a, b, c, f\} \), \( B = \{b, c, d, e, g\} \) Find:

1) \( A' \) ; \( B' \)

2) \( A \cup B \)
   
   \((A \cup B)'\)

3) \( A \cap B \)
   
   \((A \cap B)'\)

4) \( A' \cap B' \)

5) \( A' \cup B' \)

De Morgan Law:

a) \((A \cup B)' = A' \cap B'\)

b) \(A' \cup B' = (A \cap B)'\)
Example 3 Cont.: If $U = \{a, b, c, d, e, f, g, h, i\}$ and $A = \{a, b, c\}$, $B = \{b, c, d, e, g\}$. Draw the Venn diagram.
Example 4: If $U = \{a, b, c, d, e, f, g\}$ and $A = \{a, b, f\}$, $B = \{c, d, e, g\}$ Find:

1) $A \cup B$

2) $A \cap B$

Partition: a) Union is all or: $A \cup B = U$
b) Nothing in Common or: $A \cap B = \emptyset$

Example 5: Mark has two sets of courses to choose from:

Set $A = \{\text{Chemistry, Math, English}\} = \{C, M, E\}$
Set $B = \{\text{French, History, Geology}\} = \{F, H, G\}$

Find:

a) the number of courses that are in $A$ and $B$.

b) the number of courses that are in $A$ or $B$. 
Example 6: Mike has two sets of courses to choose from:

Set \( A = \{ \text{Chemistry, Math, English, History}\} = \{C,M,E,H\} \)

Set \( B = \{ \text{Math, English, French}\} = \{M,E,F\} \)

Find:

a) the number of courses that are in \( A \) and \( B \). \( n(A \cap B) \)

b) the number of courses that are in \( A \) or \( B \). \( n(A \cup B) \)

c) the number of courses that are in \( A \) only.

\[
n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]
Example 7: In a survey of 80 people, it was found that:
45 read the Sport magazine (S)
40 read the Time magazine (T)
10 read both magazines (T & S)
Find the number of people that read:

a) Time only  b) Sport only  c) neither magazine  d) either magazine
Example 8: In a survey of 200 people, it was found that:

- 150 listen to Rock music \((R)\)
- 80 listen to Slow music \((S)\)
- 55 listen to Classic music \((C)\)
- 60 listen to Rock and Slow music \((R \& S)\)
- 25 listen to Classic and Slow music \((C \& S)\)
- 40 listen to Rock and Classic \((R \& C)\)
- 15 listen to all \((R \& S \& C)\)

Find the number of people that listen to:

a) Rock only  

b) 2 kind of music  

c) Rock and Slow but not Classic  

d) none
Example 9: In a survey, it was found that:

- 55 students took History ($H$)
- 45 students took English ($E$)
- 25 students took Geography ($G$)
- 7 students took English and History but not Geography
- 5 students took Geography and History but not English
- 3 students took Geography and English but not History
- 30 students took English only

Find the number of students that took:

a) the three subjects at the same time   b) History only
Example 10: If $A$ and $B$ are subsets of $U$ and: $n(A) = 5$, $n(B') = 7$, $n(A' \cap B') = 3$. Find $n(A \cup B)$.
Example 11: Let $A$, $B$, and $C$ be subsets of $U$, use the Venn diagram to shade the solution:
Example 11 Cont.: Let $A$, $B$, and $C$ be subsets of $U$, use the Venn diagram to shade the solution:
Example 11 Cont.: Let \( A, B, \) and \( C \) be subsets of \( U \), use the Venn diagram to shade the solution:

\[
\begin{align*}
\text{i) } & (A \cap B') \cup C \\
\text{J) } & (A \cup B \cup C) \cup (A \cap B) \\
\text{k) } & (A \cup B \cup C) \cap (A \cap B)
\end{align*}
\]
Example 12: Which of the following statements is True?

a) \( A^\prime \cup B^\prime = (A \cup B)^\prime \)

b) \( A\cap B^\prime = (A \cap B)^\prime \)

c) \( A \cap B^\prime \subseteq A \cap B \)