Project 2. Functions of a Single Variable and Graphing

Objective

In this project you will learn how to define and graph a function in Maple.

Narrative

In Maple, the function \( f = f(x) \) is defined by the command:

\[
f := x -> \text{expression in } x
\]

and we can use the command:

\[
\text{plot}(f(x),x=a..b)
\]

to plot the function \( f \) from \( x = a \) to \( x = b \)

\[
\text{plot}(f(x),x=a..b,y=c..d)
\]

to plot the function \( f \) from \( x = a \) to \( x = b \), limiting output to points with \( y \)-coordinates between \( c \) and \( d \)

\[
\text{plot}([f(x),g(x)],x=a..b,y=c..d)
\]

to plot functions \( f \) and \( g \) from \( x = a \) to \( x = b \), restricting output to points with \( y \)-coordinates between \( c \) and \( d \)

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference, along with some questions you should ask yourself.

\[
> \text{# Project 2. Functions of a Single Variable and Graphing}
\]

\[
> \text{restart;}
\]

Clear Maple’s memory.

\[
> f := x -> \text{abs}(x^2-4)-2*x;
\]

Let \( f(x) = |x^2 - 4| - 2x \).

\[
> f(3);
\]

Evaluate \( f \) at \( x = 3 \).

\[
> \text{plot}(f(x),x=-50..50);
\]

Plot \( f \) from \( x = -50 \) to \( x = 50 \). (What does this graph tell you about the behavior of \( f \)? What would you say \( f(0) \) is just by looking at this graph? What value do you get when you substitute 0 for \( x \) in \( f(x) \)?)

\[
> \text{plot}(f(x),x=-1..1);
\]

Plot \( f \) from \( x = -1 \) to \( x = 1 \). (What does this graph tell you about the behavior of \( f \)? It’s not so clear anymore? You can estimate \( f(0) \), but how do you reconcile this graph with your earlier one? Well, …)

\[
> \text{plot}(f(x),x=-4..4);
\]

Plot \( f \) from \( x = -4 \) to \( x = 4 \). (Do you see what a difference the choice of \( x \)-values makes in getting a good plot of the graph of a function?)

\[
> \text{plot}(\tan(x),x=-\Pi..\Pi);
\]

Plot \( \tan(x) \) from \( x = -\pi \) to \( x = \pi \). (If you know what this graph should look like, it should be clear that this is not a good graph of the tangent function.)

\[
> \text{plot}(\tan(x),x=-\Pi..\Pi,y=-3..3);
\]

Plot \( \tan(x) \) from \( x = -\pi \) to \( x = \pi \), restricting output to points with \( y \)-coordinates between \( -3 \) and \( 3 \). (Note that we get a much better graph by restricting \( y \)-coordinates, although now we have different scales on the \( x \)- and \( y \)-axes.)

b) Continue by typing the command lines below into Maple in the order in which they are listed.
Graph $f(x) = x$ and $g(x) = 2 \sin x$ from $x = -\pi$ to $x = \pi$ on the same set of axes. (Note that we can plot more than one function on the same set of axes. Let us find the coordinates of the point $(x, y)$ of intersection of these graphs for which $x > 0$, in effect finding one of the solutions of $x = 2 \sin x$. Looking at our graph we observe that the correct $x$ value lies between 1.8 and 1.9; so we graph both functions over this interval.)

Plot the graphs of $f(x) = x$ and $g(x) = 2 \sin x$ from $x = 1.8$ to $x = 1.9$. (From this graph it appears that the correct value of $x$ lies between 1.89 and 1.90. So we graph both functions over this interval.)

Replot $f(x)$ and $g(x)$ from $x = 1.89$ to $x = 1.90$. (From this graph it appears that the correct value of $x$ lies between 1.895 and 1.896. By continuing this process of “zooming in”, we could arrive at an arbitrarily precise estimate of the correct $x$-value.)

At this point, make a hard-copy of your typed input and Maple’s responses. Then, in each of the last three plots, label by hand the graphs of $f(x) = x$ and $g(x) = 2 \sin x$ by “$f(x) = x$” and “$g(x) = 2 \sin x$”, respectively.

**Comments**

1. Another way to solve the equation $x = 2 \sin x$ is to solve the equation $x - 2 \sin x = 0$ by “zooming in” on the zero of the function $h(x) = x - 2 \sin x$.

2. Observe that the more you “zoom in” on the graph of a function, the more linear the graph of a function such as $g(x) = 2 \sin x$ appears. Do you think this is true for all functions?

3. Even though the graphs of curves may appear to be smooth, the way programs like Maple graph functions is by plotting a finite number of points and “connecting-the-dots” with short line segments. Thus, rather than seeing a smooth curve when you plot a function such as $\sin x$, you are actually seeing a polygonal approximation to its graph. This may be why you see vertical lines in the graph of $\tan x$: if $x$ is an odd multiple of $\pi/2$ then the value of $\tan x$ is undefined, but — unaware of this — Maple goes ahead and draws a line segment (from a point a little to the left of $x$ to a point a little to the right of $x$), and that segment is exactly what you see when you see the vertical lines. (To see the dots that Maple uses to graph $f$, see what happens when you use the command `plot(f(x),x=a..b,style=point`).)

4. One thing you should have learned from this project is that while Maple is very useful for graphing, you cannot trust it completely: it might *not* reveal important behavior of a function if you look at a graph of the function at the wrong scale.

5. To specify a piecewise-defined function such as

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } 0 < x \leq 1 \\ -2x & \text{if } x > 1 \end{cases}$$

in Maple use:

```maple
f := x -> if x <= 0 then x^2 elif x <= 1 then 2*x+1 else -2*x fi;
```

In addition to the `if/then/elif/else/fi` control structure, Maple offers many other structures. To learn more about them, check out the Programming section of Maple’s Help.