1 Using $\tau = \dot{v}_2 \cdot v_3$, derive the formula for a general parameterization $x = x(t)$ of a curve,

$$\tau = \frac{x'(t) \times x''(t) \cdot x'''(t)}{|x'(t) \times x''(t)|^2}.$$  

2 Prove that if all osculating planes pass through a fixed point, then the curve is a plane curve.

3 Let $x = x(s)$ be a curve with arc-length parameter. If the initial points of all the unit tangent vectors $v_1$ are shifted to the origin, their new end points trace out a curve $v_1 = v_1(s)$ on the unit sphere. This curve is called the spherical indicatrix of the curve $x = x(s)$. If $\tilde{s}$ denotes arc-length on the spherical indicatrix, show that

$$k(s) = \frac{d\tilde{s}}{ds}.$$  

4 Show that circular helices have constant curvature and constant torsion.

5 Let $x = x(s)$ be a plane curve. Then the trace of the curve

$$x^*(s) := x(s) + \frac{1}{k(s)} v_2(s),$$

is the locus of the centers of the osculating circles. Show that the tangent vector to $x^* = x^*(s)$ is in the direction of the principal normal of $x = x(s)$. 
