The symplectomorphism groups of rational 4-manifolds

Abstract. This talk is on the topology of the infinite dimensional Lie group \( \text{Symp}(M^4, \omega) \), the symplectomorphism group of a rational 4-manifold \((M^4, \omega)\). It’s a long term open problem initiated by Gromov and later intensively studied by many people. In particular, Donaldson conjectured that \( \pi_0 \text{Symp}(M^4, \omega) \), also called the symplectic mapping class group, is generated by Dehn twists along Lagrangian spheres. This conjecture was studied by Seidel, McDuff, etc. We will report some recent progress answering this conjecture in the positive, and explicitly describing \( \pi_0 \text{Symp}(M^4, \omega) \) as a spherical braid group. Also, for higher homotopy groups \( \pi_i, i \geq 1 \) of \( \text{Symp}(M^4, \omega) \), we completely determine the stable range when deforming the symplectic form. In particular, we compute the rank of \( \pi_1 \text{Symp}(M^4, \omega) \) and point out its relation to Hofer metric (the canonical Finsler metric on \( \text{Ham}(M^4, \omega) \)). This talk is based on joint works with Tian-Jun Li and Weiwei Wu.