Integration with Computer Algebra Systems

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Objective

To illustrate some practical aspects of using a computer algebra system such as Maple to perform integration.

Narrative

If you just want to evaluate an integral then Maple’s int command can take you quite far with very little effort. There are some important practical aspects of using int, however, and in this project we illustrate some of these. (While we work with Maple in this project, similar types of comments apply to other computer algebra systems, such as Mathematica, Derive, and MATLAB.)

Tasks

1. Open a new Maple worksheet and type the commands below into it. These commands are aimed at finding \( \int \frac{dx}{2x+3} \).

\[
> \text{# Your name, today's date} \\
> \text{# Integration with Maple} \\
> \text{restart;} \\
> \text{# Task 1} \\
> \text{int(1/(2*x+3),x);} \\
\]

Note that there are two problems with the answer that is returned: The first is that the absolute value signs around the argument \( 2x+3 \) are missing, and the second is that the +C is missing. (See Task 4 below.)

2. Continue by typing the commands below into Maple. They are aimed at finding \( \int x(x-1)^99 \, dx \).

\[
> \text{# Task 2} \\
> \text{int(x*(x-1)^99,x);} \\
\]

Maple evaluates this integral by expanding the integrand and then integrating the resulting polynomial term-by-term. While Maple’s answer is certainly correct, this integral can be done by a fairly simple \( u \)-substitution, and the resulting form of the answer is arguably simpler than Maple’s. (See Task 5 below.) Integals involving the trig functions — both the circular trig functions and the hyperbolic trig functions — are particularly likely to have multiple forms because of the identities that these functions satisfy.

3. Continue by typing the commands below into Maple. They are aimed at finding \( \int (1+\ln x)\sqrt{1-x\ln x} \, dx \).

\[
> \text{# Task 3} \\
> \text{int((1+ln(x))*sqrt(1-x*ln(x)),x);} \\
\]

The fact that Maple returns an answer of \( \int (1+\ln x)\sqrt{1-x\ln x} \, dx \) means that Maple is unable to evaluate this integral. Just because Maple cannot evaluate this integral does not mean that it is impossible, or even extremely difficult, to evaluate, however. (See Task 6 below.)

At this time make a hard copy of your typed input and Maple’s responses. Then, by hand:

4. Write the correct form for what you get when you evaluate \( \int \frac{dx}{2x+3} \).

5. a) Evaluate \( \int x(x-1)^99 \, dx \) using the \( u \)-substitution \( u = x - 1 \).
b) Describe two (different) ways you could use Maple to prove that your answer is correct.

6. Observe that the integral $\int (1 + \ln x) \sqrt{1 - x \ln x} \, dx$ is the integral of a product, and that one factor — namely $(1 + \ln x)$ — is the derivative of $x \ln x$. Thus we can simplify this integral — and perhaps complete evaluating it — by making the $u$-substitution $u = x \ln x$. Do this: make the substitution $u = x \ln x$ and complete the evaluation of this integral.

Comments

While the major focus of this project has been on some of the practical aspects of using computer algebra systems such as Maple to evaluate integrals, it is important to note that we are even discussing the use of computer algebra systems such as Maple to evaluate integrals to begin with. After all, today’s computers are extremely powerful and today’s programming languages are so easy to use that if our only concern were to find an extremely accurate approximation to the area under a curve or an extremely accurate approximation to the volume of a solid of revolution then we wouldn’t ever need to concern ourselves with the symbolic evaluation of integrals. Implicitly, then, there is something beyond the applications we have studied up to now that demands knowledge of integration. We will start getting to this “something” in our discussion of differential equations.