Compound Interest: 
\[ P = P_0 \left(1 + \frac{r}{n}\right)^{nt} \]

- \(P_0\): the principal, amount invested
- \(P\): the new balance
- \(t\): the time in years
- \(r\): the rate, (in decimal form)
- \(n\): the number of times it is compounded.

**Ex1:** Suppose that $5000 is deposited in a saving account at the rate of 6% per year. Find the total amount on deposit at the end of 4 years if the interest is:

a) compounded annually,
\[ n = 1: \quad P = 5000(1 + 0.06/1)^{(1)(4)} = 5000(1.06)^4 = $6312.38 \]

b) compounded semiannually, \(n = 2\):
\[ P = 5000(1 + 0.06/2)^{(2)(4)} = 5000(1.03)^8 = $6333.85 \]

c) compounded quarterly, \(n = 4\):
\[ P = 5000(1 + 0.06/4)^{(4)(4)} = 5000(1.015)^{16} = $6344.93 \]

d) compounded monthly, \(n = 12\):
\[ P = 5000(1 + 0.06/12)^{(12)(4)} = 5000(1.005)^{48} = $6352.44 \]

e) compounded daily, \(n = 365\):
\[ P = 5000(1 + 0.06/365)^{(365)(4)} = 5000(1.00016)^{1460} = $6356.12 \]

**Continuous Compound Interest:**
Continuous compounding means compound every instant, consider investment of 1$ for 1 year at 100% interest rate. If the interest rate is compounded \(n\) times per year, the compounded amount as we saw before is given by:
\[ P = P_0 \left(1 + \frac{r}{n}\right)^{nt} \]

The following table shows the compound interest that results as the number of compounding periods increases:

<table>
<thead>
<tr>
<th>Compounded</th>
<th>(n)</th>
<th>Compound amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>annually</td>
<td>1</td>
<td>((1+1/1)^1 = 2)</td>
</tr>
<tr>
<td>monthly</td>
<td>12</td>
<td>((1+1/12)^{12} = 2.6130)</td>
</tr>
<tr>
<td>daily</td>
<td>360</td>
<td>((1+1/360)^{360} = 2.7145)</td>
</tr>
<tr>
<td>hourly</td>
<td>8640</td>
<td>((1+1/8640)^{8640} = 2.71812)</td>
</tr>
<tr>
<td>each minute</td>
<td>518,400</td>
<td>((1+1/518,400)^{518,400} = 2.71827)</td>
</tr>
</tbody>
</table>

As the table shows, as \(n\) increases in size, the limiting value of \(P\) is the special number \(e = 2.71828\)

If the interest is compounded continuously for \(t\) years at a rate of \(r\) per year, then the compounded amount is given by:
\[ P = P_0 e^{rt} \]

**Ex2:** Suppose that $5000 is deposited in a saving account at the rate of 6% per year. Find the total amount on deposit at the end of 4 years if the interest is compounded continuously. *(compare the result with example 1)*

\[ P_0 = $5000, \quad r = 6\% \quad t = 4 \text{ years} \]
\[ P = 5000.e^{(0.06)(4)} = 5000.(1.27125) = $6356.24 \]
Ex3: Find the amount to be invested at a rate of 8% compounded continuously in order to get $12,930 in 6 years.

\[ P = 12,930, \quad r = 0.08, \quad t = 6 \text{ years.} \]

\$12,930 = P_o e^{(0.08)(6)} \quad \text{then} \quad P_o = \$8000

The Growth, Decline:

<table>
<thead>
<tr>
<th>Continuous Growth: ( P = P_o e^{rt} )</th>
<th>Continuous Decline: ( P = P_o e^{-rt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Growth: ( P = P_o (1 + r)^t = P_o a^t )</td>
<td>Annual Decline: ( P = P_o (1 - r)^t = P_o b^t )</td>
</tr>
</tbody>
</table>

where \( a > 1 \) \hspace{1cm} where \( 0 < b < 1 \)

Doubling Time = \( \frac{\ln 2}{r} \) (for continuous growth)

Half-life = \( \frac{\ln 2}{r} \) (for annual growth)

Ex4: The growth rate in a certain country is 15% per year. Assuming continuous growth:

a) If the population is 100,000 now, find the new population in 5 years.

b) When will the 100,000 double itself?

Ex5: In 1965 the price of a math book was $16. In 1980 it was $40. Assuming the continuous growth:

a) Find \( r \) and write the equation.


c) After when will the cost of the book be $32?

Ex6: A couple want an initial balance to grow to $211,700 in 5 years. The interest rate is compounded continuously at 15%. What should be the initial balance?

Ex7: The population of a city was 250,000 in 1970 and 200,000 in 1980 (Decline). Assuming the population is decreasing continuously, find the population in 1990.

Ex8: At what rate compounded annually will an investment of $2100 accumulate to $3400 by the end of 6 years.

Ex9: How long does it take amount to double at 8.5% compounded: annually, continuously?

Ex10. The population of a certain town is declining exponentially due to immigration. If the population was reduced by 20% after 10 years, find the decline rate.

Ex11. Write the equation of problem 10 but: If only 85% the population are present after 10 years.

Ex12. The half-life of a certain radioactive substance is 12 days. If there are 10 grams initially:

a) find the rate.

b) when will the substance be reduced to 2 grams?

Ex13. Convert the function \( P = 400 e^{0.05t} \) to the form \( P = P_o a^t \)

Ex14. Convert the function \( P = 2000(1.08)^t \) to the form \( P = P_o e^t \)

Answers: 4. (211700, 4.62) 5. (6.1%, 54.28, 11.36) 6. (100,000) 7. (160,000)

8. (8.36%) 9. (8.5, 8.15) 10. (2.23%) 11. 0.85 = \( e^{-10t} \)

12. (5.78%, 27.86) 13. \( P = 400(1.0513)^t \) 14. \( P = 2000e^{0.07696t} \)